Explaining coalitions: a taxonomy of formal models of government formation and breakdown

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Abstract

Coalition theory is one of the fields that has extensively benefited from formal modelling in political science. Since the publication of William Riker's (1962) seminal book, a variety of coalition models has been developed to explain government formation and breakdown. Nevertheless, despite models' diversity, the literature seems to judge their predictability solely in terms of empirical tests, usually deriving hypotheses from their propositions and testing via statistics. In this research note, I survey classical and more recent formal models in coalition theory, demonstrating how they serve different purposes. I build a taxonomy of models that acknowledges these purposes, and, more importantly, the roles they play in building explanations about coalitions. I identify three types of models: conceptual models, whose conceptual and theoretical value relies on the mathematical deductions entailed in the model; quasi-conceptual models, which formalise explanations of regularities in the real-world; and extrapolative models, which allow for an empirical test of formal models via a variety of statistical methods. All of these types of models coexist, generating explanations and setting research agendas.
Keywords: coalitions; formal models; rational choice theory

Explicando coaliciones: una taxonomía de modelos formales de formación y colapso de gobiernos

Resumen

La teoría de la coalición es uno de los campos que se ha beneficiado ampliamente de los modelos formal en ciencia política. Desde la publicación del libro seminal de William Riker (1962), se ha desarrollado una variedad de modelos de coalición para explicar la formación y el colapso del gobierno. Sin embargo, a pesar de la diversidad de los modelos, la literatura parece juzgar su capacidad de previsión únicamente en términos de testes empíricos, generalmente derivando hipótesis de sus proposiciones y testándolas a través de estadísticas. En esta nota de investigación, examino modelos formales clásicos y más recientes en la teoría de la coalición, demostrando cómo sirven a diferentes propósitos. Construyo una taxonomía de modelos que reconoce estos propósitos y, lo que es más importante, los roles que desempeñan en la construcción de explicaciones sobre coaliciones. Identifico tres tipos de modelos: modelos conceptuales, cuyo valor teórico y conceptual se basa en las deducciones matemáticas que contiene el modelo; modelos cuasi-conceptuales, que formalizan explicaciones de regularidades en el mundo real; y modelos extrapolativos, que permiten una prueba empírica de modelos formales a través de una variedad de métodos estadísticos. Todos estos tipos de modelos coexisten, generando explicaciones y estableciendo agendas de investigación.
Palabras-clave: coaliciones; modelos formales; teoría de la elección racional

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Introduction

As an academic endeavour, coalition theory mobilises concepts and models from different fields in political science. One cannot think of coalitions without resorting to the language of party competition, electoral systems, veto players and ministerial cabinets. Indeed, coalitions are all about putting these pieces together. Game theory and spatial models constitute the essence of modelling in coalition theory, answering questions about the mechanisms underlying the formation, duration and breakdown of governments.

These models have become pervasive in the literature on coalition theory. The current state of affairs reveals an immense accumulation of knowledge in terms of mathematical models, conceptual analyses and empirical tests. Rational choice models play a crucial role in this process, by setting agendas, framing research questions, advancing concepts and explaining empirical regularities. They serve distinct purposes, not only the widely-spread view that tends to relate a model's predictions to an exercise of data-fit. The essence of modelling is the constant search for explanation, the unravelling of mechanisms and the generation of predictions about general phenomena. In this sense, this paper aims to provide an understanding of models that acknowledges the different purposes for which they are designed. It departs from the following question: what are the distinguishing characteristics of formal modelling in coalition theory? I argue that coalition models serve different purposes, generating explanations of various kinds and models that should not be judged by the single metric of empirical testability. The novelty of this approach consists in the surveying of classical and recent developments in coalition theory, for it is a field where at least three types of models can be identified: conceptual models, whose conceptual and theoretical value relies on the mathematical deductions entailed in the model; quasi-conceptual models, which formalise explanations of regularities in the real-world; and extrapolative models, which allow for an empirical test of formal models via a variety of statistical methods (regression, multivariate analysis, maximum likelihood estimation etc.). Each class of model tailors explanations of coalition formation and breakdown in its own fashion, therefore my goal in this paper consists in developing a novel taxonomy of models that identifies the different ways they generate explanation and prediction.

The research agenda on coalition theory prospered after the publication of William Riker's *The Theory of Political Coalitions* (1962), even though much of the spatial modelling was already in place thanks to the works of Harold Hotelling, (1929), Duncan Black (1958) and Anthony Downs (1957), not to mention earlier works by John von Neumann and Oskar Morgenstern (1953) on the minimum-winning hypothesis and its posterior test by William Gamson (1961). Riker devised the concept of minimal-winning coalition, which consists of coalitions as large as necessary to ensure winning (Hindmoor and Taylor, 2015: 85). Politicians see the process of acquiring support and votes as costly, and hence they optimise their actions by guaranteeing precisely the number of votes necessary to approve their proposals. In other words, Riker's sees the problem of coalition-formation as an optimisation problem: the equilibrium resides on the optimal point of minimal costs and minimal size.

Riker's model was intrinsically simple and its main insight rests on the concept of minimum-winning coalition. However, such simplicity could not explain the variation observed in the real-world. Some claimed that disequilibrium was more frequent than equilibrium, linking this idea to McKelvey-Schoffield chaos theorem. Riker (1980: 443) conceded to this: "Disequilibrium, or the potential that the status quo be upset, is the characteristic feature of politics". Yet instead of dismissing the initial model, political scientists and economists have attempted to find causes for disequilibrium and alternative explanations to the variation observed in the real-world (Dowding, 1995: 44-48). A myriad of models have been built to understand the role of parties and the

formateur (Bassi, 2013; Bäck and Dumont, 2008; Diermeier and Merlo, 2004; Diermeier and Vlaicu, 2011), the process of coalition formation and breakdown (Ansolabehere et al., 2005; Giannetti and Sened, 2004; Martin and Stevenson, 2001 and 2010; Martin and Vanberg, 2005; Volden and Carrubba, 2004), and the relationship between ministers and coalitions (Huber and Martinez-Gallardo, 2008; Laver and Shepsle, 1996). These models typically address the roles played by prime-ministers, parties, institutional setting, legislative behaviour, ministers etc. The recent institutionalist turn in political science helped to bind these agendas together into a consistent research framework (Dowding and King, 1995: 1-4; Dowding, 1995: 50-55). Nonetheless, instead of following one single theoretical and empirical path, coalition theory is rather an umbrella that covers a myriad of models and research designs, which involve formal modelling, historical analysis, comparative method, statistical tests, and computational simulations.

In the sections that follow, I present some examples that sustain the aforementioned categories of models and allow for an evaluation of what we have learnt from models of coalition formation and breakdown. Perhaps in this field one can fully understand how models play various roles in the discipline. More importantly, coalition theory shows how relevant conceptual and quasi-conceptual models are to building an understanding of general phenomena without resorting to direct statistical testing. Evidently, it also draws attention to the challenges of performing tests, conducting experiments, and measuring variables. Furthermore, the vast literature on comparative political coalitions reminds us how empirical tests that lack an underlying explanatory model are bound to accumulate knowledge without building firm explanations. This does not mean that researchers should resign from testing, but rather that formal models are necessary to confer meaning to statistical findings.

The papers is divided into four sections. The first section introduces conceptual models in coalition theory, highlighting their centrality to the foundations of the theoretical endeavour in this literature. In the second section, I discuss the role played by quasi-conceptual models in tailoring explanations of empirical regularities observed in real-world coalitions, such as Gamson's conjecture. The third section focuses on extrapolative models of coalition formation and breakdown, which are paramount to the empirical testing of hypotheses. An assessment section follows suit, and I thereby summarise the taxonomy of formal models in coalition theory.

Conceptual models: setting the foundations of coalition theory

The roots of coalition theory can be traced back to the seminal works of John von Neumann and Oskar Morgenstern, and Lloyd Shapley, who developed the conceptual models that allowed Riker to design his minimal-winning coalition model. Coalitions are the outcome of bargaining processes that are productively represented by the tools of game theory. In the *Theory of Games and Economic Behaviour* (1953), von Neumann and Morgenstern set the ground for Shapley's model, which, according to Roth (1988: 4), "summarize[s] the complex possibilities facing each player in a game in characteristic function form by a single number representing the 'value' of playing the game".

The simplicity of Shapley's (or Shapley-Shubik value, in its further developments) model rests on the fact that with three straightforward axioms, one can reach a formula that allows to evaluate an actor's centrality in a coalition: "Our definition of the power of an individual member depends on the chance he has of being critical to the success of a winning coalition" (Shapley and Shubik, 1988: 41). As general it is, the model offers a concept applicable to any situation where one needs to determine the power of an actor. Some general results of coalition profiles in congress chambers and the United Nations Security Council illustrate how the Shapley-Shubik value is central to the understanding of coalition formation and functioning.

This conceptual model of coalition theory has set the foundations for later developments in the theory. The Shapley-Shubik value served primarily is a groundbreaking model upon which other scholars not only tested its explanatory power, but also derived hypotheses that have been scrutinised in further research projects. Riker (1962), for instance, devised his concept of minimum-winning coalition based on Shapley's model. Clearly, in this case, a conceptual model was essential to set a new research agenda, which produced other models of different natures. To be sure, conceptual models allow for the derivation of empirically testable hypotheses, although they by themselves are not supposed to be directly tested.

To the extent that conceptual models set research agendas and the theoretical/ epistemological lenses through which researchers analyse political phenomena, they play the fundamental role of drawing scholars' attention to quintessential elements of these very phenomena. In the case of coalition theory, the Shapley-Shubik value has defined the importance of pivotal actors in coalition formation and functioning. The conceptual leap to Riker's model is direct and clear. Nevertheless, conceptual models in coalition theory were not capable of explaining regularities in real-world coalitions, nor they were prone to testing in more complex settings. Even Riker's model was subjected to criticism for its inability to explain why disequilibrium in coalition formation and maintenance ocurred, especially when comparing across multiples cases (Dowding, 1995). Therefore, conceptual models of coalition do not suffice to offer overarching explanations, opening an window of opportunity for other approaches to modelling.

Quasi-conceptual models of coalition formation: explaining regularities

In political science, scholars are frequently faced with patterns in real-world phenomena. The existence of patterns suggests that some sort of mechanism must be at operation. Ideally, political scientists would propose descriptions of the mechanism, further testing them in order to explain the phenomena. Nevertheless, across political science one may find various examples of patterns which are known to exist (due to empirical evidence), but which are not fully described as part of a theoretical endeavour. In coalition theory, Gamson's law – which states that governments distribute portfolios in proportion to each party's contribution of seats to the coalition (Carroll and Cox, 2007; Gamson, 1961) – illustrates the case of an empirical regularity lacking an explanatory mechanism.

This sort of epistemological problem has led coalition modellers to devise quasi-conceptual models. This type of model is designed to explain an observed empirical regularity by resorting to mathematical deductions. Data come first and the model explains their patterns by unravelling potential explanatory mechanisms. To illustrate quasi-conceptual models, I shall present two different ones in coalition theory, showing how the literature uses logic and mathematical tools to enhance arguments and tailor explanations about coalition formation and breakdown.

In a recent article, Michael Laver and Kenneth Benoit (2015) develop a model-cum-typology of party systems that account for the variations of coalition governments observed in 29 European parliamentary democracies. They call this classification "the basic arithmetic of legislative decisions", for it represents the possible outcomes of the electoral process that lead parties to power; and the outcomes of the bargaining process within the legislature. Table 1 displays their typology.

Table 1: Universe of possible legislative party systems

Single winning party	*No single winning party*
	$S_1 < W$

<table>
<tr><td rowspan="3">$S_1 \geq W$</td><td colspan="3">$S_1 + S_2 \geq W$</td><td rowspan="3">$S_1 + S_2 < W$</td></tr>
<tr><td colspan="2">$S_1 + S_3 \geq W$</td><td rowspan="2">$S_1 + S_3 < W$</td></tr>
<tr><td>$S_2 + S_3 < W$</td><td>$S_2 + S_3 \geq W$</td></tr>
<tr><td>Single winning party</td><td>Strongly dominant party</td><td>Top-three</td><td>Top-two</td><td>Open</td></tr>
</table>

Source: Adapted from Laver and Benoit (2015: 277). Notation: S_i is legislative party i and W is the winning quota to successfully pass proposals.

The classification in Table 1 represents different scenarios under which coalitions can form. It is not simply a classification per se, because Laver and Benoit (2015) consider the implications of simple logical statements to draw conclusions about how coalitions should form. Those implications are not derived in a game-theoretical fashion, but they still resort to basic mathematical tools and numerical examples to prove the argument and sustain the model. Once their model-cum-typology is presented, Laver and Benoit (2015) proceed to analyse the empirical distribution of those classes of party systems across their dataset of European democracies. Furthermore, they also resort to multinomial logistic regression to estimate the level of change of each legislative type in face of changes in seat shares. Essentially, their model attempts to provide conceptual grounds to describe and explain regularities observed in European parliaments and estimate coalition change of each type.

Anna Bassi (2013) follows a similar procedure in her works on government formation. Bassi is concerned about what she calls a "prominent empirical regularity": "the share of cabinet portfolios that each government party receives is almost perfectly proportional to the share of legislative seats it contributes to the government (…), with no evidence of a formateur advantage even when portfolio payoffs are weighed by salience" (Bassi, 2013: 777). Departing from that evidence, which sustains Gamson's conjecture of proportional portfolio allocation, she develops a game-theoretical model that "makes predictions in line with Gamson's Law" (Bassi, 2013: 778). Essentially, her bargaining model determines endogenously the role of the formateur, and it follows four stages of bargaining, which are solved for equilibrium via backward induction. A similar effort to solve the puzzle of Gamson's law has been systematically conducted by Indridi Indridason (2015), who has developed a game-theoretical approach coupled with empirical testing to tailor explanations to the puzzle.

Both models depart from empirical regularities to solve a theoretical puzzle in the literature. A great deal of the literature on coalition theory has developed upon the failures of previous models that could not account for specific phenomena in the real world. Even Riker's initial theory was subjected to criticisms for the lack of predictive power, especially when confronted with empirical data.[2] However, these failures have led researchers to explore the causes and mechanisms of the divergence between models and the real-world. In Bassi's and Laver and Benoit's cases, their models offer explanations of certain regularities in data, connecting the dots through mathematical expressions, implications and propositions. Without the models, we would only be able to observe patterns in data; perhaps, some statistical tests would be capable of correctly predicting phenomena of their interest based upon recurring patterns, but we would not be able to tell the difference between a good and a bad prediction because an underlying explanatory mechanism would be

[2] Riker's model was a simple implementation of the Shapley-Shubik value to determine the minimal-winning coalition. It was institution-free and, therefore, could not account for the variation observed in real world cases. Many counter-examples contradicted his theory on the surface, if one cares about folk predictions. However, Riker's model showed that coalition formation is not a random nor obvious process, which is a prediction of the type-level (i.e., a prediction about a general phenomenon). By devising the minimal-winning coalition concept, Riker identified a rationale for government formation and breakdown, which allowed for further theoretical and empirical developments.

missing. In this sense, the value of their models resides in the explanatory nature and how they give meaning to observed patterns. In other words, they accommodate data into an overarching quasi-conceptual model.

In order to make this point clear, it is worth mentioning the example of conservation laws in physics. Scientists have always known that some physical quantities obey conservation laws – Isaac Newton's classical mechanics is based upon the notion that momentum is always conserved. Energy conservation was an essential part of thermodynamics and it had been validated by countless empirical evidence and tests (Kuhn, 1977). Nevertheless, a mathematical model capable of explaining why some quantities are always conserved was still absent until the beginning of the 20th century. In 1915 and 1918, mathematician Emmy Noether deduced a mathematical model that proved the connection between conservation laws and symmetries, a feat of such importance to physics that Nina Bayer (1999) sums up in the following terms: "They [Noether's theorems] led to a deeper understanding of laws such as the principles of conservation of energy, angular momentum, etc., and also were instrumental in the great discoveries of gauge field symmetries of the 20th century". Empirical evidence was finally connected with a formal, explanatory model that bridged two distinct domains in physics and mathematics.

The aforementioned coalition models are analogous to Noether's theorem: they have been designed to provide explanations of regularities observed in the real world, but which lacked an explanatory mechanism. They are quasi-conceptual because their goal consists in providing explanations to patterns in data rather than solely testing them or advancing a theoretical argument. Daniela Giannetti and Itai Sened's (2004) coalition model of the Italian parliament follows similar lines, with a particular way of connecting the model's predictions with empirical data. They resort to visual tools (mostly graphs of left-right vs institutional dimensions) to locate parties and coalitions throughout Italian history, relating their positions in the two-dimensional space to predictions in the model. They do so because they express doubts about the prospects of using statistical tests (namely, regression) to validate models. Giannetti and Sened (2004: 513) say:

> An important part of the failure to use mathematical models in the analysis of real-life politics stems from an undue loyalty to traditional quantitative statistical analysis in the study of politics. Regression analysis in whatever form is unlikely to help much in analysing such complex environments as multiparty parliamentary systems. The abstract mathematical models of this reality make it succinctly clear.

I would not make such a strong statement, for there are prospects for combining statistics and formal models (Ansolabehere et al., 2005; Signorino, 1999 and 2003). Furthermore, researchers might be interested in testing the outcomes of a given model, rather than the underlying assumptions and structure. This is a methodological decision, which does not necessarily end in failure. In the next section, I shall turn to coalition models that are tested via statistics.

Testing coalition models

The bulk of the literature in coalition theory is empirically-oriented, and much of this orientation assumes the form of statistical tests and models. Since the publication of Riker's model, political scientists have been testing his predictions, as well as other models' predictions, via statistics and computational simulations.

Laver and Shepsle (1996) made an important contribution to the testing of coalition models by developing their own model and conducting a computational simulation to validate its structure before fitting empirical data into it. The underlying assumption in their model consists in shifting

the attention to the allocation of ministries to explain government formation (and breakdown). Laver and Shepsle (1996: 14-15) state:

> For us, then, a *government* consists of an allocation of authority in particular policy jurisdictions to particular political parties with well-known policy representations in the areas. Because there is only a handful of key policy jurisdictions and only a limited number of parties with ministerial-calibre politicians who can credibly be nominated to these, the numbers of different potential cabinets is also limited. Thus, government policy outputs are selected from a finite set of policy forecasts, each forecast being associated with a particular portfolio allocation. The finite nature of the set of credible potential governments means that the business of building and maintaining a government is explicable (…) in a more straightforward manner than the general spatial model suggests.

The portfolio allocation model is grounded in eight assumptions based upon Rational Choice Theory (henceforth, RCT), and unfolds over three stages. Firstly, a party is selected to form a cabinet, proposing a certain allocation of ministries. If such allocation differs from the status quo, it advances to stage two, where the members of the coalition accept or reject the proposal. If they accept, it proceeds to the third stage, where the cabinet is voted in the chamber; if members reject, it returns to the first stage.[3] In their analysis, two concepts are crucial: the equilibrium cabinet, which "once it is formed, stays formed because no political actor with the ability to act in such as to bring down the cabinet and replace it with some alternative has the incentive to do so" (Laver and Shepsle, 1996: 61); and the strong party (S), which "participates in every cabinet preferred by a majority to the cabinet in which Party S takes all portfolios" (Laver and Shepsle, 1996: 70). Very strong parties lead to the formation of equilibrium parties, because it has the ability to shift the bargaining closer to its ideal point.[4]

The spatial model in Laver and Shepsle (1996) was derived using measures of distance in Euclidean space and theorems based on set theory. The authors acknowledge that formal models "yield expressions that, while rigorously derived, do not give us much intuition about what is likely to happen in particular real-world cases" (Laver and Shepsle, 1996: 93), and resort to computational simulation to explore the predictions of their model, specifically "of which factors affect the existence and identity of a strong party" (Laver and Shepsle, 1996: 97).

The portfolio allocation model as developed by Laver and Shepsle is representative of the empirically-oriented works in coalition theory. Formal models are tested via the predictions entailed in their theorems and propositions. Perhaps, one of the clearest examples of this approach is the seminal paper on government formation by Lanny Martin and Randolph Stevenson (2001), where the authors derive 21 hypotheses from coalition models and subject them to a test of maximum-likelihood estimation (MLE). They select the MLE because it "allows the researcher to choose a distribution for the dependent variable that is appropriate to the true form of that variable" (Martin and Stevenson, 2001: 38), which is an important concern if one wants to perform a test that adequately represents the structure of the formal model. Many other researchers follow the same lines: Volden and Carrubba (2004) derive a series of variables to test five coalitions models (including their own) using time-series cross-sectional analysis of a dichotomous dependent variable; Diermeier and Merlo (2004) test for different bargaining procedures that are frequently assumed in formal models; Martin and Stevenson (2010) use the conditional logit model to test for

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[3]Evidently, there exist variations across parliamentary systems, yet the model is fairly representative of the general procedures in government formation.

[4] In their analysis, the authors conclude: "[A] powerful position in the decisive structure enhances a party's control over the making and breaking of governments in two ways. First, a more dominant position in the decisive structure is far more likely to make a party strong, and hence an essential member of any government. Second, while even parties with weaker positions in the decisive structure can be strong if they occupy the right position in the configuration of party positions, dominant parties are far more likely than these to be very strong, and thus not to have to rely on their ability to win standoffs" (LAVER and SHEPSLE, 1996: 105).

the impact of incumbency in government formation; Becher and Christiansen (2015) develop a formal model of dissolution threats issued by the prime minister and their effects in legislative bargaining, resorting to content analysis to measure the threats, and then combining the measurements with public opinion and legislative data in standard logit regressions.

Alternative empirical tests of formal models can derive statistical measures directly from the structure of the mathematical component of the model. This the case presented in Ansolabehere et al. (2005), where the authors identify a misspecification problem in the literature. According to them, the bulk of tests of coalition models uses the number of seats as a measure, but the formal models to which they refer are derived from assumptions on voting weights. As they state (Ansolabehere et al., 2005: 552):

> Voting weights complicate empirical testing of these models. Seat shares do not equal voting weight share, and (…) the approximation can be quite poor. As a result, regression analyses relating seat shares to shares of posts, as done in most empirical work on this topic, will generally yield biased estimates of the relationship between voting weights and cabinet posts. The estimated coefficients of other variables, such as an indicator of the formateur, will also be affected.

These considerations are important not only to their model, but also to the tailoring of explanations and conclusions from the empirical test. The authors, thus, propose a model where the costs and voting weights can be regressed. They do so, because "the appropriate independent variable that measures a party's bargaining strength is its share of the voting weight in the legislature" (Ansolabehere et al., 2005: 554). It is worth noting that this is one among many other possible specifications to the problem. The authors are aware of that and the implications of other specifications, but they have chosen this particular one because they were able to estimate the model. This a crucial issue, since models (whether purely mathematical or statistical) are constrained by their tractability and solvability (Signorino, 2003).[5]

Despite such innovativeness, the literature still follows the standard approach of generating hypotheses from models' theorems and testing them independently from the structure. Hanna Bäck and Patrick Dumont (2008), in designing their two-stage model of the role of the formateur in government formation, provide a compelling argument about why the structural approach might be difficult to represent. The authors (Bäck and Dumont, 2008: 360-361) state:

> Optimally we would like to statistically model the two stages as interdependent, e.g., using a similar approach of analyzing strategic interaction as presented by Signorino (1999). Problems of application in this setting however abound: first, we are here dealing with a two-stage game where one actor (e.g., the Head of State) is making a choice in the 1st stage, and another actor (the formateur) is then interacting with other actors (the other parties); second, we are dealing with a large number of choice alternatives, and the number of choices vary across formation opportunities; finally, we are here dealing with some theories that make multiple equilibrium predictions, which makes it difficult to assign probabilities over outcomes (Signorino 1999, p. 294). An alternative statistical approach would be to use some sort of nested model, which would allow for sequential choices.

As we can see, deriving a test that respects the structure of the original formal model is not an easy task. Nevertheless this is a matter of uttermost importance for the validation of empirical tests and their conclusions. Explanation rests on the mathematical derivations entailed in the model, for they provide the links between the operating mechanisms of a particular phenomenon. Furthermore, from the same model different researchers might generate different hypotheses, but telling which one is true to the model is a matter that can only be settled by referring to the structure. If the test is poorly connected to the structure, then settling the case becomes much

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[5] This is particularly true of phenomena pervaded with nonlinearities, which in politics take the form of uncertainties. These uncertainties, once represented in a formal model, may produce divergent solutions, depending on how they manifest themselves in the process of solving for the model.

harder. Therefore, building this connection – for example, via appropriate derivation of equations (see Signorino, 2003) or measurements (Ansolabehere et al., 2005) – is essential to enhance explanations and conclusions.

Assessment

Writing in their seminal book *Games and Decisions* (1957), Robert Luce and Howard Raiffa addressed the challenges that coalition studies would face as part of the conceptual and empirical features of coalitions. As a game that is played by *n* actors, it is only natural that the level of complexity entailed in a model is far higher than that of a two-person game. The authors (Luce and Raiffa, 1957: 156) suggest:

> A major obstacle to developing a satisfactory theory of coalition formation is that in the present formalizations of a game no explicit provisions are made about communication and collusion among the players (…). Thus any theory of collusion, i.e., of coalition formation, has a distinctly *ad hoc* flavour. The difficulties in making assumptions about communication appear, at least superficially, to stem from the variety of rules which are found in empirical situations. (…) In addition to the conceptual complications of collusion, there are inherent practical complications as *n* gets larger, for the number of possible coalitions increases at a fantastic rate; the difficulty of a detailed analysis of a two-person game such as chess is minor compared to a similar analysis of most *n*-person games. One of the principal features of the current theory is to bypass such a detailed analysis. That we can successfully avoid combinatorial problems at the conceptual level does not necessarily mean that we can do so when dealing with empirical situations.

Since then, we have walked a long path of modelling and testing. Even though the specificity of many studies seem to confirm the aforementioned *ad hoc* character of the literature on coalition theory, this is an impression limited to the surface of the theoretical and empirical advances in the field. The difficulties posed by a n-player game have been tackled through the theoretical and methodological lenses within specific institutional settings. The rules of the game eliminate combinatorial problems that would render coalition models intractable. Furthermore, thanks to an enormous collective effort to collect, systematically organise and test data, researchers can tailor their models to explain observable patterns and derive their hypotheses.

The three classes of models presented above reflect the lessons learned since the publication of Riker's book. Conceptual models have set the theoretical grounds that allowed for the development of empirical tests – not to mention data collection – and have led to a better understanding of the boundaries of explanation in coalition theory. Regularities in data which lack explanatory mechanisms have extensively benefited from quasi-conceptual models. Last, but not least, statistical tests have played an important role in assessing coalition models' predictions. All these models combined constitute the edifice of coalition theory.

However, when it comes to testing formal models in coalition theory, one important challenge still remains: how should modellers and empiricists alike connect the mathematical part of the formal model and the statistical test? This issue has recently been placed under scrutiny by Signorino and his colleagues, who have been developing solutions to the problem os structural consistency between formal model and statistical test. Bridging both sides of the same problem demands returning to the original deductions entailed in the mathematical model and statistical equations, connecting them via mathematical tools such as Taylor series, numerics etc. Evidently, this is a particular set of solutions which requires specific training, meaning that the bulk of empirical testing will probably follow the path of deriving hypotheses from a model's outcomes and testing them via classical statistical tests.

Nonetheless, the taxonomy hereby proposed reveals the relevance of mathematical models – and rational choice models, in particular – to political science, which is of uttermost importance if

one considers the misunderstandings and prejudices against RCT. To be sure, after the publication of Donald Green and Ian Shapiro's critique (1994), many scholars have echoed criticisms against the explanatory potential of rational choice models. Some have argued that rational choice theorists were plotting against other methodological approaches, attempting to confer a hegemonic status on RCT (Becker, 1976). Others have suggested, following Green and Shapiro (1994), that rational choice models are intrinsically trivial, failing to explain various cases and political phenomena. Although some of those criticisms might be true of certain rational choice models, it is rather a generalisation that fails to understand the diversity of rational choice approaches (Cox, 1999 and 2004; Dowding, 1995, 2005, 2016; Hindmoor and Taylor, 2015; Morton, 1999), not to mention the current declining presence of formal models in the pages of top-tier journals (Jacoby et al., 2017; Ishiyama, 2015; Norris, 1997). In this sense, the explanation-oriented taxonomy of rational models offers a more nuanced understanding of modelling and models, and their relevance to the discipline.

Among many things, coalition theory has taught us that formal models serve distinct purposes, each of which answering its own set of research questions. The existence of the aforementioned classes of models – which is just one among others that could be tailored to classify models – contradicts the discourse that labels models as fables or parables (Cartwright, 2010; Rubinstein, 2012). Models as fables might be comparable to conceptual models, but not all models are solely concerned with concepts, paradoxes and other theoretical issues. In coalition theory, models play all three roles that I have identified, and each shape the research agenda in their own respect. In doing so, they contribute to the accumulation of knowledge in coalition behaviour and the development of the field as a whole. Table 2 summarises the main ideas advanced by the aforementioned taxonomy of models.

Table 2: Taxonomy of models

Type	Description	Examples
Conceptual	These models advance concepts and predictions via mathematical expressions derived from set theory and game theory. They are not empirically testable, yet their predictions offer logical explanations about general phenomena.	· Lloyd Shapley (1988) · Richard D. McKelvey (1976) · Norman Schoffield (1978)
Quasi-conceptual	The model explains an observed empirical regularity by resorting to mathematical deductions. Data come first and the model explains their patterns.	· Anna Bassi (2013) · Daniella Giannetti and Itai Sened (2004) · Indridi Indrason (2010 and 2015) · Michael Laver and Kenneth Benoit (2015)

Extrapolative	*Data-fit*	Mathematical model and statistical test are not structurally linked via mathematical expressions. Hypotheses are formulated based on the model's propositions and theorems, and then subject to an appropriate statistical test.	· Craig Volden and Clifford Carrubba (2004) · Hanna Bäck and Patrick Dumont (2008) · Lanny Martin and Randolph Stevenson (2001) · Michael Laver and Kenneth Shepsle (1996)
	Mathematical-statistical	Statistical tests are derived directly from the mathematical model. In this case, the test represents the details of the model. There is a structural, mathematical link between the formal model and the statistical test.	· Stephen Ansolabehere et al. (2005)

Source: Authors' work. *Note:* The examples provided here refer only to coalition theory. The classification, however, can be extended to other models.

Nevertheless, there are still challenges to be faced. As mentioned previously, addressing the structural settings of models is an issue of uttermost importance, which may enhance the explanatory capabilities of empirical tests. Yet this requires political scientists to combine efforts to devise appropriate mathematical-statistical links between model and test, not to mention measurements, which are an integral part of model testing. Measuring the correct variables in the model guarantees that the test is true to the mathematics, and it is itself a challenge. Our measurements are based upon data available, and on our interpretations of what they mean to the model and statistical test. However, they may prove to be incorrect, inaccurate or imprecise, as shown in the case of voting weights. Therefore, extra efforts to improve measurements constitute an important challenge in coalition theory. Nonetheless, by being aware of these methodological issues, we might be able to work on inventive ways for collecting appropriate data and building the necessary bridges between model and test.

Conclusion

Since the publication of Riker's book, the field of coalition theory has thrived and now constitutes one of the main research areas in political science. Throughout the field's development, RC models played an important role in tailoring explanations and generating predictions about a variety of phenomena involving coalition formation and breakdown. These models shed light on the connections between agents, institutional rules and political contexts, offering insightful perspectives about the dynamics of coalitions.

Throughout this paper, I have surveyed examples of coalition models attempting to show how they serve different purposes when building explanations. Despite much of the literature's (opponents of models included) tendency to think of a model's predictability solely in terms of empirical tests, models can come in distinct flavours, offering predictions in the conceptual level; unravelling the explanatory and predictive mechanisms underlying regularities; and extrapolating the mathematical expressions to fit data or derive statistical tests. All these classes share in common the essential characteristic of models, i.e., predictability. After all, models are designed to generate

predictions and explanations, and that should be the currency to judge their success. The question here is that prediction should not be restricted to the results of an empirical test, as I hope to have demonstrated in this paper.

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