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Maximizing the information and validity of a linear composite in the factor analysis model for continuous item responses

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This paper develops results and procedures for obtaining linear composites of factor scores that maximize: (a) test information, and (b) validity with respect to external variables in the multiple factor analysis (FA) model. I treat FA as a multidimensional item response theory model, and use Ackerman's multidimensional information approach based on maximum likelihood (ML) estimation of trait levels. This approach, when applied to the FA model, leads to particularly simple results as far as maximizing test information is concerned. Developments concerned with validity appear to be new, and I use ML results in the context of error-in-variables regression. Graphical procedures for representing both type of results are proposed. The developments are illustrated with two empirical examples in personality measurement.

When using multidimensional instruments that measure related dimensions it is sometimes useful to obtain a single test score that represents the respondent. For example, in a multidimensional questionnaire that measures different facets of anxiety it can be of interest to obtain a single, general anxiety score. This score is generally obtained as a linear composite that has optimal properties in some sense. The topic of finding 'best' linear composites has been widely researched in classical test theory, and the most usual composites are weighted combinations of the raw scores that maximize either the reliability, or the validity with respect to external variables (e.g. Gullicksen, 1950 chap. 20, Wang and Stanley 1970).

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In multidimensional item response theory (MIRT) the problem of determining an optimal linear composite has been approached from the theory of maximum likelihood (ML) estimation and the related concept of test information. Ackerman (1994, 1996) proposed an index of multidimensional test information as well as a general approach for determining a linear composite in the trait space that maximizes the test information. Ackerman's approach is equivalent to determining a linear composite whose ML estimate (MLE) has minimal conditional standard error of measurement (SEM). So far, Ackerman's approach has been used in relation to standard nonlinear IRT models, and the results are relatively complex because in these models the information or the SEM varies generally with the trait levels. As far as the writer knows, the approach has not been considered in relation to external validity.

In typical-response (personality and attitude) measurement the most common type of item format is the graded response format, such as the Likert scale (Dawes, 1972, Ferrando, 2002, Hofstee, Ten Berge and Hendricks, 1998). Furthermore, more continuous formats such as line segments, feeling thermometers, visual analogues, etc. are more and more used with the increasing use of computerized administration (Ferrando, 2002). By far, the most common model for analyzing this type of items is linear factor analysis (FA). Although, FA is a model for continuous-unlimited variables and can not be strictly correct for item responses (who are to a greater or lesser degree discrete and bounded) I take the position that FA is so widely used because it behaves as a reasonably good approximation in most practical applications (e.g. Atkinson, 1988, Hofstee, Ten Berge and Hendricks, 1998).

Most applied psychometricians tend to consider FA as disconnected from IRT models. However several authors have emphasized the correspondence between them. In particular, McDonald (e.g. 1999) provided a general framework in which FA and most IRT models are particular cases of a general latent trait model described by a strong principle of local independence. This framework is generally used for treating IRT models as nonlinear FA models. In this paper, however, we shall take the reciprocal approach and we shall consider multiple FA as a MIRT model. By using this approach, we shall show that Ackerman's developments for maximizing information in a linear composite become particularly simple in the FA case. Next we shall extend the ML approach to determine the linear composite that maximizes validity with respect to an external variable. Finally the results derived will be illustrated using two empirical examples in personality measurement.

The Basic Results

In this section we shall present the general results that are needed for the developments in the following sections. The results will be described using both scalar and matrix notation. For the sake of simplicity and didacticism we shall present the scalar results in relation to the bidimensional FA model. The matrix results are generalizations which are valid for any number of dimensions.

For a respondent *i* and an item *j,* the structural model for the multiple FA is

$$X_{ij} = \mu_j + \lambda_{j1}\theta_{i1} + \lambda_{j2}\theta_{i2} + \varepsilon_{ij} \,. \tag{1}$$

In matrix notation:

$$\mathbf{x_i} = \boldsymbol{\mu} + \boldsymbol{\Lambda}\boldsymbol{\theta}_\mathbf{i} + \boldsymbol{\varepsilon}_\mathbf{i} \tag{2}$$

For fixed $\boldsymbol{\theta}$**,** the item responses are distributed independently (local independence). The conditional distribution is assumed to be normal, with mean and variance given by

$$E(X_j \mid \theta) = \mu_j + \lambda_j\theta_i \quad ; \quad Var(X_j \mid \theta) = \sigma^2_{\varepsilon j} \tag{3}$$

were the conditional mean is the linear item response function of the model. These results imply the well-known covariance structure

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Phi}\boldsymbol{\Lambda}' + \boldsymbol{\Psi} \tag{4}$$

were $\boldsymbol{\Phi}$ is the inter-factor correlation matrix, and ψ is the diagonal matrix containing the residual variances $\sigma_{\varepsilon j}^{2}$.

Under the assumption of conditional normality above, it follows that the MLEs of the trait levels for respondent *i* are the Bartlett estimated factor scores (Mulaik, 1972). In matrix notation they are obtained as

$$\hat{\boldsymbol{\theta}}_\mathbf{i} = (\boldsymbol{\Lambda}'\boldsymbol{\Psi}^{-1}\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}'\boldsymbol{\Psi}^{-1}(\mathbf{x_i} - \boldsymbol{\mu}) \tag{5}$$

Now, by using standard procedures, the information matrix (Kendall and Stuart, 1976, chap. 18) is found to be

$$I(\theta_1,\theta_2) = -E\begin{bmatrix} \dfrac{\partial^2 LnL}{\partial\theta_1^2} & \dfrac{\partial^2 LnL}{\partial\theta_1\partial\theta_2} \\ \dfrac{\partial^2 LnL}{\partial\theta_1\partial\theta_2} & \dfrac{\partial^2 LnL}{\partial\theta_2^2} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n}\dfrac{\lambda_{j1}^{\;2}}{\sigma_{\varepsilon j}^2} & \sum_{j=1}^{n}\dfrac{\lambda_{j1}\lambda_2}{\sigma_{\varepsilon j}^2} \\ \sum_{j=1}^{n}\dfrac{\lambda_{j1}\lambda_2}{\sigma_{\varepsilon j}^2} & \sum_{j=1}^{n}\dfrac{\lambda_{j2}^{\;2}}{\sigma_{\varepsilon j}^2} \end{bmatrix} = I\,. \tag{6}$$

And the asymptotic covariance (**ACOV**) matrix of the MLEs of **θ** is given by the inverse of **I** (we shall assume that **I** is positive definite). In particular, the square roots of the values in the main diagonal of **ACOV** are the asymptotic SEM of the factor score estimates for the corresponding factor. The key result for the present procedures is that the elements of **I,** and so of **ACOV** do not depend on **θ.** This means that the SEM of each set of factor scores is constant. Finally, we note that asymptotic in this context means as the number of items become arbitrarily large.

Maximizing Information

Consider now the problem of finding the linear composite of the factors that is measured with the maximum accuracy that can be attained. We can define this composite in a given angular direction of the θ space, by considering the weights of the composite as the direction cosines

$$\theta_{cI} = \beta_1\theta_1 + \beta_2\theta_2 \tag{7}$$

so that $\beta^2_1 + \beta^2_2 = 1$. For any number of factors and in matrix notation:

$$\theta_{cI} = \mathbf{b'\theta} \quad , \quad \mathbf{b'b} = 1 \tag{8}$$

By standard ML theory, it follows that the MLE of θ_{cI} is the linear composite of the MLEs of θ_1 and θ_2 (Kendall and Stuart, 1976). Asymptotically its conditional variance is given by the quadratic form

$$Var(\hat{\theta}_{cI} \mid \theta_1,\theta_2) = \beta_1^2 Var(\hat{\theta}_1 \mid \theta_1,\theta_2) + \beta_2^2 Var(\hat{\theta}_2 \mid \theta_1,\theta_2) + 2\beta_1\beta_2 Cov(\hat{\theta}_1,\hat{\theta}_2 \mid \theta_1,\theta_2) \tag{9}$$

where the variances and covariances are the corresponding elements of **ACOV**. In matrix notation, the quadratic form is

$$Var(\hat{\theta}_{cI} \mid \boldsymbol{\theta}) = \mathbf{b'}\,\mathbf{ACOV}\,\mathbf{b} \quad (10)$$

and the inverse of the scalar quantity (10) is Ackerman's (1994) multidimensional information measure. The weights that maximize the information of the linear composite are thus those that minimize the conditional variance in (10), i.e. the SEM of the MLE of the linear composite. The conditions for obtaining the weights are

$$Minimize: \mathbf{b'}\,\mathbf{ACOV}\,\mathbf{b} \quad subject \quad to: \quad \mathbf{b'}\,\mathbf{b} = 1. \quad (11)$$

This is a standard problem of finding the extrema of a quadratic form subject to restrictions (e.g. Basilevsky, 1983). The vector of weights **b** is the eigenvector of **ACOV** associated to the smallest eigenvalue, and this eigenvalue is the conditional variance of the MLE of the linear composite.

It is of interest to compare this simple result with the results obtained in nonlinear standard MIRT models. Because in linear AF the elements of **ACOV** do not depend on **θ**, once a linear composite in a given direction has been specified, the MLE of this composite will have a constant SEM. In contrast, in standard MIRT models the elements of **ACOV** do depend on θ. So, even when a direction has been defined, the SEM of the MLE will be different at each point along this direction. This means, in turn, that the direction of maximum information must be obtained at each point in the θ space. Usually maximum information in standard MIRT models can only be assessed using graphical procedures such as clamshell plots or directional plots (Ackerman, 1994, Reckase and McKinley, 1991).

In bidimensional and tridimensional FA solutions the results so far obtained can also be graphically represented in a form similar to the information plots used with nonlinear MIRT models. For the bidimensional case, θ_1 and θ_2 are taken as coordinate axes, and the linear composite is represented as a vector in this coordinate system, with direction given by the direction cosines, and length equal to the multimensional information (or some suitable function of it). This plot allows us to assess visually: (a) the direction in the θ space of the single test score that provides the best measurement, (the direction of the vector) and (b) the precision of this measurement (the length of the vector).

Maximizing External Validity

Consider now an external variable y (e.g. a criterion or a different test) that is theoretically related to θ_1 and θ_2. We want to obtain the linear composite that maximizes the value of the validity coefficient with respect to y.

$$\theta_{cV} = \alpha_1\theta_1 + \alpha_2\theta_2 . \tag{12}$$

So that the product-moment correlation between y and θ_{cV} is as large as possible. If the 'true' trait levels or factor scores were known, this would be a standard multiple regression problem. However, the true trait levels are not known, and what we have in this case are the MLEs of θ_1 and θ_2, (i.e. Bartlett's factor scores). We can consider these estimates as proxies for the 'true' trait levels, and for a generic factor g, write

$$\hat{\theta}_g = \theta_g + \omega_g . \tag{13}$$

If the MLE estimates in (13) are used in place of the unknown true values, the situation becomes an error-in-variables multiple regression. If the problem of substituting proxies for true values is ignored and standard multiple regression procedures are used, the estimated weights are likely to be biased, and the direction of the bias is unpredictable (Bohrnstedt and Carter, 1971).

Error-in-variables estimation in multiple regression is complex, but it becomes notably simplified if the proxies are well-behaved and met some desirable conditions (Johnston, 1972 chap. 9). In the present context, these conditions are: (a) The measurement errors ω_g have zero expectation, (b) The measurement errors ω_g and the true levels θ_g are linearly independent, and (c) Var(ω_g) is known. We shall now show that, asymptotically, the ML factor scores met the three conditions.

First, MLE are asymptotically unbiased (Kendall and Stuart, 1976). So, from (13) we have

$$E(\hat{\theta}_g \mid \theta_g) = \theta_g \quad so, \quad E(\omega_g \mid \theta_g) = 0 . \tag{14}$$

And it follows that the marginal expectation of the errors is zero, which is condition (a). Furthermore, if the errors have zero expectation at any trait level, the regression slope of ω_g on θ_g is flat, which implies that ω_g and θ_g are linearly independent (condition 2). Finally the variances (and covariances) of the error terms are known (condition 3): they are the corresponding elements of **ACOV**. Now, if we denote by $\mathbf{S_{ML}}$ the covariance matrix between the estimated factor scores, and by $\mathbf{s_{MLy}}$ the vector containing the covariances between the estimated factor scores and *y*, then, the error-in-variables unbiased estimate of α, (i.e. the vector of weights that maximize validity), is given by (Johnston, 1972).

$$\hat{\boldsymbol{\alpha}} = (\mathbf{S_{ML}} - \mathbf{ACOV})^{-1}\mathbf{s_{MLy}}. \tag{15}$$

As in the previous section, the linear composite that maximizes validity can be represented as a vector in a given direction in the θ space. In this case, however, the length of the vector depends on the characteristics of the external variable *y*. What I propose is to scale this vector using the same criteria as in the previous section, so that it can be represented together with the linear composite that maximizes information. More in detail, I propose to represent the maximum validity composite as a vector in the same coordinate system as above, with the obtained direction, and with length equal to the value of the multidimensional information provided in this direction. The plot which displays simultaneously both vectors: maximal information and maximal validity is essentially a clamshell plot, and provides us clear and quick information regarding (a) the extent to what the directions of measurement for the composite that maximizes information and the composite that maximizes validity are similar, and (b) the amount of precision of both sets of scores.

Illustrative Examples

The results so far discussed are illustrated using two empirical examples in the personality domain. The first example used a Spanish version of Buss and Perry's (1992) aggression questionnaire (AQ; Andreu et al., 2002, Morales-Vives et al. 2005, Condon et al. 2006) as the main measure. The AQ is a multidimensional questionnaire made up of 5-point Likert items that measures four related dimensions of aggression. For the present illustration I chose the subscales that measure physical aggression (PA, 9 items), and anger (AN, 6 items). As external measure I used the dysfunctional impulsivity (DI) scale scores of the Spanish version of Dickman's impulsivity inventory (Chico et al. 2003). According to the theory, DI is positively related to aggression (Vigil-Colet and Codorniu, 2004). The measures were administered to a sample of 241 secondary school students between 12 and 17 years old. Data was kindly supplied by Dr. A. Vigil.

The bidimensional FA model in (1) was fitted to the 15 AQ items using the Jöreskog-Howe specifications (Howe, 1955, Jöreskog, 1979). Item 1 was used as a marker for PA and item 10 as a marker for AN. With this specification the estimated interfactor correlation was ϕ=0.47. The model was fitted using robust ML estimation as implemented in the Mplus program (Muthén and Muthén, 1999). The fit was reasonably good. The root mean squared error of approximation (RMSEA) and its 90% confidence interval were 0.04 and (0.03; 0.06), respectively, and the Non-Normed Fit index was 0.97. The parameter estimates of the model are shown in table 1

As table 1 shows, the solution is far from approaching an independent-cluster structure. However, the factors can be reasonably well distinguished, and generally tend to agree with the subscale labels. The first factor, which is mainly defined by the PA items, is defined by more indicators and has a clearer structure. The second factor, which is more related to the AN items, is less well defined and has a more complex structure.

Next I used the results in table 1 and obtained the information matrix according to (6) and its inverse the **ACOV**. The matrices are shown below

$$\mathbf{I} = \begin{bmatrix} 7.42 & 0.75 \\ 0.75 & 1.89 \end{bmatrix} \quad ; \quad \mathbf{ACOV} = \begin{bmatrix} 0.14 & -0.05 \\ -0.05 & 0.55 \end{bmatrix}$$

From the results above it follows that the SEM for the MLE of θ_1 and θ_2 are respectively: *sqrt*(0.14)=0.37, and *sqrt*(0.55)=0.74. Clearly, the estimated factor scores in the PA factor are far more accurate than the AN factor scores, which agrees with the solution in table 1. The eigenvalues of **ACOV** are 0.13 and 0.56, and the eigenvector associated to the smallest eigenvalue is [0.99, 0.13]. So, the linear composite that maximizes information in this example is

$$\theta_{cI} = 0.99\theta_1 + 0.13\theta_2$$

The conditional variance is the value of the smallest eigenvalue: 0.13, and the SEM of the MLE of the composite is *sqrt*(0.13)=0.36, which is smaller than the SEM of each individual MLE. Ackerman's multidimensional information is 1/0.13=7.69.

Figure 1 shows the maximum-information linear composite (labelled as *MaX I*). It is represented by a vector that starts at the origin and whose length is the square root of the multidimensional information: *sqrt*(7.69)=2.77. As expected, the vector tend to fall more along the θ_1 axis, lying in a narrow angle of about 7º with respect to this axis. Overall, the contribution of the more precise PA factor to the linear composite that maximizes information is far larger than the contribution of the AN factor. In other words, the composite resembles far more the PA factor than the AN factor.

Table 1. Bidimensional factor solution for the AQ items. First example

Item	θ_1	θ_2	σ^2
PA1	0.80	0.00	1
PA2	1.16	-0.40	0.78
PA3	0.95	-0.26	1.39
PA4	0.60	-0.02	0.83
PA5	1.05	-0.33	1.08
PA6	1.08	-0.17	0.81
PA7	-0.44	0.65	1.82
PA8	0.67	-0.03	0.92
PA9	0.65	0.22	1.34
AN1	0.00	0.66	1.82
AN2	0.31	0.21	1.39
AN3	0.36	0.66	1.01
AN4	0.32	-0.22	1.61
AN5	0.38	0.39	0.88
AN6	0.42	0.58	1.04

We turn now to validity results. The weights for the linear composite that maximizes validity with respect to DI scores were obtained by using equation (15) and found to be 0.77 and 0.47. So, the linear composite that maximizes validity in this example is

$$\theta_{cV} = 0.77\theta_1 + 0.47\theta_2$$

Using these weights, a respectable validity coefficient of 0.39 was obtained. The disattenuated validity coefficient by correcting only the MLE's (i.e. the estimated validity if the unknown true factor scores were used instead of Bartlett's estimates) was 0.42. For the sake of comparison, the validity coefficients obtained with the separate factor scores were 0.35 (PA factor) and 0.26 (AN factor).

The *Max Val* vector in Figure 1 shows the maximum-validity linear composite. It is noted that the direction of measurement of the two vectors is not the same. They are separated by an angle of about 23º. The length of the *Max Val* is the square root of the multidimensional information in this direction, and its value is 2.22, which is sensibly smaller than the length of the *Max I* vector. Overall, the results show that (a) if validity with respect to DI is to be maximized, the contribution of both factors must be now more balanced, and (b) because both vectors have different directions, to maximize validity implies to lose some precision with respect to the maximum attainable precision. (this point can be assessed by comparing the lengths of both vectors).

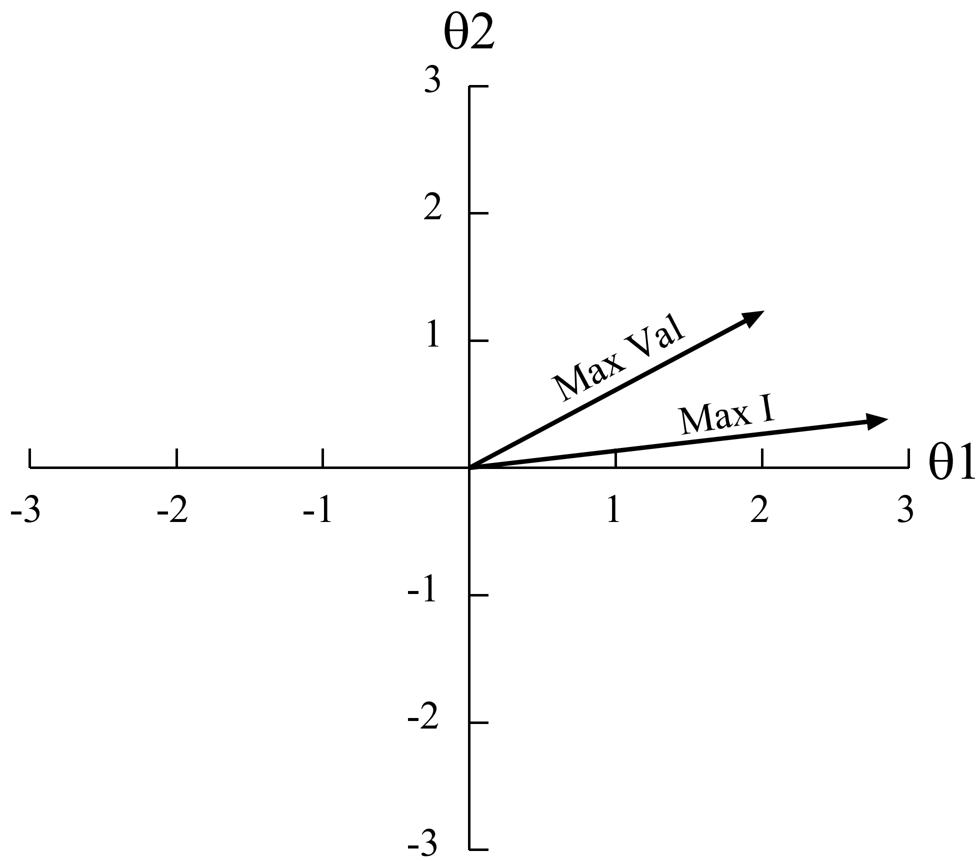

Figure 1. Graphical representation of the linear composites. First example.

The second empirical example illustrates the used of some of the present procedures in the tridimensional case. The used measure was a Spanish version of the Perceived Stress Scale (PSS; Cohen, Kamarck and Mermelstein, 1983), a theoretically unidimensional inventory made up of 14 items with a 5-point Likert format that measures the degree to which situations in one's life are perceived as stressful. The PSS was administered to a group of 203 undergraduate students.

The aim of the original research was to assess the potential impact of acquiescence and social desirability (SD) response bias on the PSS scores. To do so, an orthogonal unrestricted solution in three factors (content, acquiescence and social desirability) was obtained by using a procedure developed by Ferrando, Lorenzo-Seva and Chico (2007). Details about the procedure, goodness of fit, and resulting orthogonal solution are given in the referred article.

In a well-designed instrument, the impact of the response biases would be low, and the scores would mainly reflect the content dimension that the test intends to measure. Here we shall show how the present procedures can be useful for addressing this point. By using the FA solution in three dimensions, the estimated information matrix and the **ACOV** were found to be:

$$\mathbf{I} = \begin{bmatrix} 7.75 & -0.30 & -0.33 \\ -0.30 & 2.95 & 0.06 \\ -0.33 & 0.06 & 0.17 \end{bmatrix} \quad ; \quad \mathbf{ACOV} = \begin{bmatrix} 0.14 & 0.01 & 0.27 \\ 0.01 & 0.34 & -0.10 \\ 0.27 & -0.10 & 6.42 \end{bmatrix}$$

where the dimensions are given in the order: content, acquiescence and SD. From the results above it follows that the SEM for the MLEs are: *sqrt*(0.14)=0.37, (content), *sqrt*(0.34)=0.58 (acquiescence), and *sqrt*(6.42)=2.53 (SD). The estimated factor scores in the content factor (perceived stress) are thus far more accurate than the response-bias factor scores, which is a good result. In particular, the low information provided by the social desirability scores suggests that the impact of SD on the PSS is very low.

The smallest eigenvalue of **ACOV** is 0.12, and the corresponding eigenvector is [0.99, -0.06, -0.04]. So, the linear composite that maximizes information in this second example is

$$\theta_{cI} = 0.99\theta_{content} - 0.06\theta_{acq} - 0.04\theta_{SD}$$

The conditional variance is the value of the smallest eigenvalue: 0.12, and the SEM of the MLE of the composite is *sqrt*(0.12)=0.35, which is only slightly smaller than the SEM of the content scores. Ackerman's multidimensional information is 1/0.12=8.33.

Figure 2 shows the maximum-information linear composite as a vector in the tridimensional space. For simplicity the vector has been represented in the positive sector and, as in the first example, its length is the square root of the multidimensional information: *sqrt*(8.33)=2.88. The vector is almost collinear with the content axis, and lies in a narrow angle of about 4º with respect to this axis. So, the linear composite that maximizes information is almost identical to the content factor, and this suggests that the impact of the response biases on the PSS scores is very low. The most accurate scores that can be obtained from the PSS solution are almost pure measures of the content dimension.

Discussion

This paper treats multiple FA as a linear MIRT model, and develops procedures for determining a single test score, obtained as a linear composite of the individual factor scores, which is optimal in some sense. In particular I consider two optimization criteria: maximizing information and maximizing validity with respect to an external variable. It is found that, for the multiple FA model and for MLE of the factor scores (i.e. Bartlett's scores) both maximization problems have a closed, relatively simple solution. Furthermore, I propose a graphical representation procedure (essentially a clamshell plot) that can be useful in applications. From an applied point of view, the composite that maximizes information will possibly be more useful in individual assessment, when what is needed is a single score that discriminates among respondents as accurately as possible (perhaps for classification purposes). The maximum validity composite will be more useful in criterion-related studies, both for making individual predictions and for assessing the relative importance of the individual trait levels with respect to the criterion.

As far as the writer knows, the developments proposed here are original. However it is indeed acknowledged that they are based on well-known results. It is well known that FA can be treated as a linear IRT model, and also well-known are the asymptotic properties of MLE of the trait levels in the context of IRT models. The application of Ackerman's multidimensional information approach to the FA case is quite direct, and so is the adaptation of the graphical procedures. On the other hand the validity results are obtained by combining standard error-in-variables-regression results with standard asymptotic results on ML estimation. However, in spite that all of the proposals are relatively straightforward, I have been unable to find a single study in which the general procedures proposed here were proposed or used.

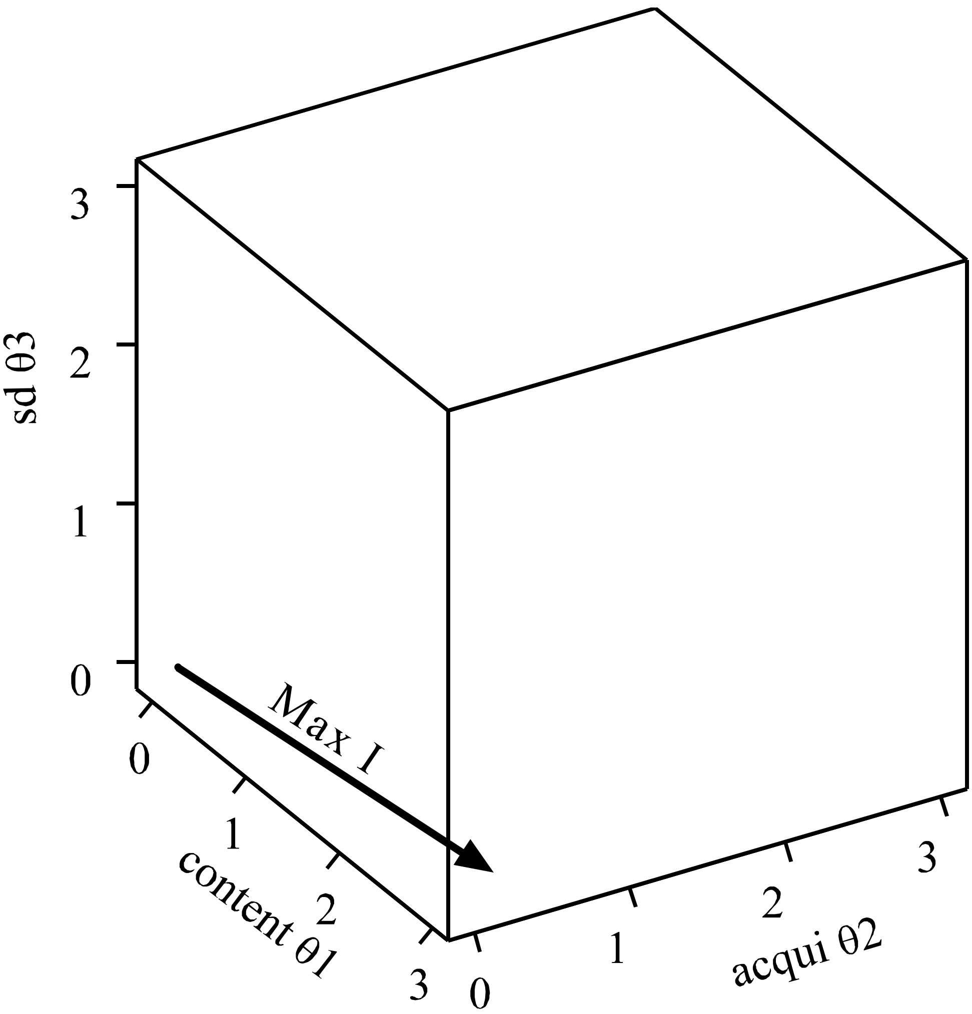

Figure 2. Graphical representation of the maximum-information linear composite. Second example.

All the procedures I propose in the article can be computed from a standard FA output by using simple routines written in matrix programs such as MATLAB (1999). However, for the present proposal to have a minimal chance to be used in applications, an user-friendly program that implements all the procedures including also the graphic displays must be available for the applied researchers. This is an objective for future research.

RESUMEN

Combinaciones lineales que maximizan la información y la validez en el modelo factorial para ítems continuos. En este artículo se desarrollan procedimientos para obtener combinaciones lineales de las puntuaciones factoriales que maximizen: (a) la información del test y (b) la validez externa en el modelo de análisis factorial múltiple. El modelo factorial se considera como un modelo multidimensional de teoría de respuesta al ítem, y esto permite utilizar el enfoque de Ackerman para la medida de información multidimensional basada en la estimación máximo verosímil de los niveles en el rasgo. Dicho enfoque lleva a resultados notablemente sencillos en lo que respecta a maximizar la información. Los resultados y procedimientos relacionados con la maximización de la validez externa parecen ser nuevos, y se obtienen desde el enfoque de la regresión con errores en las variables. Se proponen procedimientos para representar gráficamente los resultados, y se presentan dos ejemplos empíricos para ilustrar la metodología propuesta.

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