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Artículo

Price sensitivity as a measure of living standards in late-colonial Mexico city

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ABSTRACT

We exploit crop price data and seasonal climatic conditions, measured by tree-ring growth, to analyze food consumption in the last decades of colonial rule. We estimate the elasticity of substitution between wheat and maize —a measurement of price sensitivity that provides a demand-based approximation to living standards. The estimates place household expenditures between the two alternatives found in the literature: the Laspeyres (or fixed) basket and the least-costly (or "cheap") basket, albeit the estimates are closer to the latter. A fixed basket would require a household to spend on average 29% more than a "cheap" basket; the average excess expenditure for our estimates is close to 2%. The results confirm the decreasing trend on living standards in the literature, and highlight the importance of household optimizing behavior in the assessment of living standards: economic pressure and necessity elicit adaptation.

La sensibilidad a los precios como medida del nivel de vida en la Ciudad de México: 1741-1812

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RESUMEN

Analizamos el nivel de vida en el periodo colonial tardío utilizando los precios del maíz y trigo y las condiciones climáticas estacionales, medidas por el crecimiento de los anillos de los árboles. Para ello, estimamos la elasticidad precio entre el maíz y el trigo, una medida de sensibilidad a los precios que nos ofrece una aproximación a los niveles de vida por el lado de la demanda. Las estimaciones sitúan el gasto de los hogares entre las dos alternativas encontradas en la literatura: la cesta de Laspeyres (o fija) y la cesta menos costosa (o "barata"), aunque las estimaciones se aproximan más a esta última. Una cesta fija exigiría a un hogar gastar en promedio un 29% más que una cesta "barata"; las estimaciones implican un gasto medio 2% mayor para evitar la cesta barata. Los resultados confirman la tendencia decreciente en el nivel de vida en los estudios sobre el tema, y ponen de relieve la importancia del comportamiento optimizador de los hogares para evaluar su nivel de vida: la presión económica y la necesidad dan lugar a la adaptación.

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The ability of households to substitute between different foodstuffs can provide a cushion in times of food shortages. Appleby (1979), for example, shows that the portfolio of crops in England allowed the English to avoid famine by relying on weather-robust crops in times of adverse climatic conditions, while the French were unable to do so. The diversification of agricultural production allowed households to substitute across crops and reduce the impact of adverse price increases that resulted from low crop yields. Consequently, the standard of living fell more in France than in England in such times.

Most of the research on living standards assumes a Laspeyres, or fixed, basket of goods, i.e., a policy where consumption quantities are constant and do not respond to price changes. Thus, an extreme form of habit persistence determines the consumption patterns of households (Allen *et al*., 2012; Arroyo-Abad *et al*., 2012; Challú and Gómez-Galvarriato 2015). In doing so, researchers are unable to incorporate how households respond to price changes when evaluating living standards. An alternative approach minimizes food expenditure, subject to some nutritional requirements, which yields a least-costly, or "cheap", basket of goods (Allen 2001; Allen *et al*., 2011; Zegarra 2011, 2020). However, while households change their consumption when the order of prices changes, the "cheap"-basket assessment does not incorporate information from price changes that do not change the order of prices.[1]

In this article, we estimate the price sensitivity of households between maize and wheat in Mexico City for the late colonial period. We conceptualize price sensitivity using the elasticity of substitution, which quantifies how much the demand ratio changes when the price ratio increases by 1%.[2] The estimation of the elasticity of substitution provides a demand-based approximation to the price sensitivity and a measurement of the economic and nutritional well-being of Mexico City dwellers in the late colonial period. Our analysis highlights the importance of considering preferences and market prices when assessing historical living standards.

Our main contribution is the estimation of an elasticity-based, calibrated consumption policy. Our preferred estimated consumption policy matches many relevant data moments. More importantly, we locate such a consumption policy between the two (extreme) alternatives found in the literature: the "cheap" basket, which satisfies caloric necessities by substituting out the more expensive food items, and the fixed basket, which keeps the consumption quantities fixed, regardless of absolute and relative price changes. Both, the fixed and the "cheap" basket, can be applied without calibration or estimation to obtain living standards. Hence, these methods are robust to estimation errors and the availability or unavailability of data required for a calibration. However, the differences in living standards between the two policies can be considerable. The average excess expenditure of the fixed over the "cheap" basket is about 29.2% in our period of investigation, yet the decade-wide average reached 40% in the 1760s.

To estimate the elasticity of substitution we lay out a simple dynamic model economy grounded on historical evidence. Maize is planted in the first half of the year and harvested in the second half. Wheat, on the other hand, is a winter crop planted in the second half of the year and harvested in the subsequent spring. Hence, crop planting and harvesting rotate in half-yearly cycles. *Haciendas*, our producers, decide in each half-year how much land to commit to growing the seasonal crop. Climatic conditions in the subsequent half-year determine crop yields and, jointly with crop prices, the revenue from land use. Haciendas can store each crop for one period, which provides an insurance mechanism and smooths consumption. Households purchase the crops, and prices reveal their preferences. In this way, the elasticity of substitution is determined by the co-movement of climatic conditions and crop prices.

To calibrate the dynamic model, we employ historical, daily price data from Florescano (1969) for maize and from García Acosta (1988) for wheat. Given the sparse and unreliable data on consumption or harvesting volume we rely on tree ring data as a proxy for climatic conditions. We also undertake a robustness exercise using quantity information, yet we conclude that unreported sales/purchases and home production introduce an under-reporting bias.

Unsurprisingly, we find that the consumption of basic food staples responds to relative prices. Our estimates of the elasticity of substitution are all around 2 which suggests that the consumption ratio $(C = C_m/C_w)$ changes by 2% if the price ratio $(P = P_w/P_m)$ changes by 1%. Further, an elasticity of substitution above one implies the goods are substitutes while an elasticity of substitution below one suggests they are are complements.[3] At the extreme, goods are perfect complements if the elasticity of substitution is equal to zero while an elasticity that approaches infinity implies the goods are perfect substitutes.[4] Thus, the elasticity of substitution reflects the price sensitivity between two goods.[5] Our preferred estimation, which employs the Empirical-Likelihood principle, suggests the elasticity is larger than 1 by any conventional level of significance. Hence, maize and wheat are (imperfect) substitutes.

Our estimations based on the elasticity reveal household expenses that fall between the two extremes of a fixed basket and a "cheap" basket, albeit the estimated expenditure policy leans more towards the "cheap" basket. Our estimation could

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[1] This would imply that households only consume the cheapest good.

[2] Mathematically, the elasticity of substitution between maize and wheat is $\epsilon = (\partial C/\partial P)(P/C)$ where C denotes the demand for maize divided by the demand for wheat ($C = C_m/C_w$) and P the price for wheat in terms of maize $(P = P_m/P_w)$. The elasticity divides the marginal change in relative demand due to a price change, by the average ratio. The normalization leaves the concept robust against nominal changes in the measurements, such as currency or quantity measurements. Elasticities are, thus, "unit free" measures.

[3] A price increase in a complementary good leads to less consumption of both goods.

[4] A (classical) example of perfect complements is tires and cars: regardless of their relative prices, a consumer requires four tires for each car. Hence, a relative price change might alter the absolute consumption quantities, but it will not change the relative consumption quantities.

[5] There are two misunderstandings that can easily arise when dealing with the elasticity of substitution. First, the elasticity of substitution highlights the effect on relative demand from changes in relative prices. Simultaneously doubling prices might reduce the demand for both goods, and the demand might even change in relative terms. But the elasticity of substitution measures how relative demand changes when prices change in relation to each other. Second, while the elasticity of substitution can depend on the (real) income of consumers, as we discuss below, it solely measures the response of relative demand to relative price changes.

result in the fixed basket or the "cheap" basket as potential outcomes. If the elasticity of substitution is close to zero ($\epsilon=0$), the fixed basket would provide a suitable approximation for household spending. Conversely, if the elasticity is large and goods are close substitutes, a "cheap" basket would be considered more appropriate for assessing living standards. Throughout, we employ the example of a household of four where each member of the household requires 1,940 calories daily, as documented by Arroyo-Abad *et al.* (2012).[6] We contrast three household spending policies, assuming a family of four requires 7,760 calories daily: (1) a fixed-basket consumption policy consisting of a fixed fraction of maize (δ) and wheat ($1-\delta$); (2) a "cheap"-basket consumption policy such that households only consume the cheapest crop; and (3) a policy that adjusts relative consumption to relative price changes according to our (three) estimates of the elasticity.[7]

The quantitative differences across policies are relevant for the assessment of living standards. The "cheap" basket consumes, on average, about 29.2% of the available income for our period of investigation, which starts in 1741 and ends in 1812.[8] The fixed basket, on the other hand, requires 36.4% to achieve the same 7,760 calorie count. Our three estimations of the elasticity of substitution are similar in size and predict that our exemplary household spends about 29.8% of the available income. Hence, our estimations show that the fixed basket grossly overstates expenditure. The excess expenditure (over a "cheap" basket) is about 29%, while the policies based on our estimations are closer to 2%.

In addition, the fixed basket policy understates inflationary pressure. The inflation rates from our estimated consumption policies are more than twice the rate from using a fixed basket. The alternative of assuming households simply buy cheap, on the other hand, also understates inflation as the crop which is usually cheaper (maize) shows a higher inflationary trend overall. Further, the volatility of year-on-year inflation also varies across policies. Households mostly consume maize with the "cheap" basket, and a single price series is, generally speaking, more volatile than a composition of two price series. Hence, the standard deviation of inflationary rates with the "cheap" basket is higher than in the fixed basket. The volatility of our calibrated policy is close to that of the "cheap" basket. This highlights that optimizing behavior can also contribute to inflationary volatility as consumption quantities change between periods.

In terms of how living standards varied over decades, we find the highest expenditure in the 1780s, which includes *el año del hambre* in 1785-1786. Our elasticity-based estimations predict that 38.9% of the available income is spent on crops in that decade. Daily prices predict an extreme outlier in *el año del hambre* when even the "cheap" basket policy spent 84.8% of the available income on crops alone. The 1760s, on the other hand, were plentiful. The estimated policies predict about 20.7% of the available income was spent on food in that decade.

Our results confirm previous studies about falling living standards in the late colonial period. Challú and Gómez-Galvarriato (2015) document a steep drop in real wages beginning in 1770: according to their bare-bones price index, real wages decreased by 50% between 1760 and 1815. Arroyo-Abad *et al.* (2012) and Allen *et al.* (2012) document a similarly steep drop in real wages but that starts somewhat earlier, around 1760. Studies of biological well-being also find a decline in male heights that starts in the mid-century but the drop is sharpest after 1760 (Challú, 2010). The demand for textiles also falls in the last decades (Knight 2002, p. 217). In line with these results, we find the last three decades —the 1780s, 1790s, and 1800s— to require the most expenditure on food.[9]

We proceed as follows. Section 1 introduces the theoretical model and contrasts the model assumptions with the historical context. Section 2 introduces our data sources and describes the data preparation. Section 3 presents our main estimations while Section 4 discusses the implications of our results for living standards. Section 5 concludes.

1. Model economy

In order to estimate the elasticity of substitution between maize and wheat, we lay out a simple dynamic model economy with households as consumers and haciendas as producers. We first discuss the model assumptions that are critical for our inference and contrast them with the historical evidence on grain production, the storage of grains, consumption patterns, and market structure in late colonial Mexico City. Then, we present the formal model.

1.1. Assumptions and their historical context

The model is based on the assumption that a single household represents all households reasonably well.[10] While Mexico City was populated by natives, descendants of Spanish settlers, and people of mixed ancestry (mestizos),[11] García Acosta (1989) documented that all sub-populations consumed both maize and wheat. In particular, although mestizos and Indians were probably consuming more maize than Spaniards, they had also adopted bread into their diet (García Acosta, 1989, pp. 26-29).[12] Importantly, there was no sizable part of the population consuming only one crop. Hence, we deem it appropriate to employ a single consumer that represents the average consumer and the average consumer consumed both grains and substituted among them.[13]

[6] Note that all caloric intake is satisfied with crops in our example. We do this for illustrative purposes.

[7] We let δ vary for the fixed-basket policy. Our estimates place δ at around 0.74 to 0.75, depending on the estimation. However, quantitative predictions vary little given the range of δ.

[8] An unskilled construction worker earned about 3 reales a day (Challú and Gómez-Galvarriato 2015).

[9] See Table 5 in the Appendix for details on each policy and decade.

[10] This "representativeness of a single agent" is standard in most models in macroeconomics, industrial organization, monetary economics, and international trade.

[11] The censuses, undertaken throughout the 18th century, indicate that almost half of the population in Mexico City was of Spanish descent (born in Spain or creole) by the end of the century. Indians and mestizos, on the other hand, comprised close to 25% and 20%, respectively (García Acosta 1989, p. 21).

[12] Maize was consumed also for purposes other than making *tortillas*, e.g., to feed mules and pigs.

[13] An alternative specification, emphasizing the heterogeneity and relative size of any sub-populations, would introduce more preference parameters and expand the data requirements markedly.

Further, the preferences of this representative household are constant over time. There is evidence of migration by rural mestizos to urban areas in the last years of colonial rule in response to the bad harvests and increasing land inequality. Migration could change the preferences of a representative household over time. However, the consumption preferences of mestizos are likely a blend of the other two groups so that their migration, arguably, does not dramatically change the preferences of the representative household.

Households optimize their consumption choice in every period and adapt their choices given market prices and preferences. The model does not assume, by itself, that the preferences of the representative household lead to an adaptation of consumption choices to market prices. The representative household could have an elasticity of substitution that is equal to 0 which would imply that the consumption ratio does not change when the price ratio changes. However, this case is not borne out by the data. Further, the representative household in our model has a constant elasticity of substitution (CES) utility function, which assumes the elasticity of substitution, ϵ, is constant across income and price levels as well as over the estimation window.

We label our representative producer "Hacienda", but not all wheat and corn were grown in haciendas. Indian communities and small landowners also grew both, corn and wheat (Challú 2007, p. 135) which suggests that, first, these producers diversified their crop portfolio, and, second, members of these communities consumed both crops. There is also evidence that these smaller communities were integrated in market exchanges as they brought their harvest to the market, especially in years of abundant harvest (Florescano 1969, pp. 26-27). We hold the production function and the storage technology constant over the estimation window. This is in line with Knight (2002, p. 220) who argues that colonial Mexico experienced no major agricultural innovations.

In the model, the production of maize and wheat occurs in different seasons, namely, maize is planted in the first half of the year and harvested in the second half. Wheat, on the other hand, is planted in the second half of a year and harvested in the first half of the subsequent year (Gibson, 1967). Haciendas are profit-maximizing and do not myopically produce the same quantity of each crop year in and year out. Location-specific attributes, such as water supply, local soil variation, and other terrain-specific features that determine the exposure to the weather can vary so that growing more of a specific crop becomes more difficult. This applies in particular to the mountainous terrain surrounding Mexico City. Therefore, agricultural production is only extended when the hacienda finds it profitable.

We also assume that the supply of grains is local so that the land use choice of local haciendas and the local climatic conditions are the only relevant variation in the local supply of maize and wheat. There are several arguments that support this assumption. Transportation costs were high. The supply of corn was mostly from Chalco near Mexico City because of its quality and closeness (Florescano, 1969, pp. 26-27). Only in years of high prices was it profitable for agricultural producers to bring grain into the city from far away (Challú 2007, p. 77). In addition, the Spanish Crown regulated trade within New Spain and *alcaldes mayores* (local magistrates) could prohibit trade between districts. It is not until 1765 that the Real Pragmática decreed free trade across regions. This regulation may have increased the integration of local markets, but it was not fully implemented in Mexico until 1789 (Brading, 1971) and its biggest impact appears to have been to switch power from local to higher-level viceregal authorities (Challú, 2013).

Market quantities clear in equilibrium and crops can be stored. For simplicity, we assume that both crops can only be stored for half a year and that longer-term storage of crops has a negligible impact on market prices. It is challenging to preserve maize for longer periods because of its high moisture content. In particular, maize is vulnerable to warm weather and poor drying conditions. Challú (2007, pp. 123-131) discusses the difficulty of storing corn and links it to the high volatility of corn prices at the time. Rather, wheat and, to a lesser extent, regional trade helped smooth the food supply in bad years (*ibid.*, p. 80).

Challú (*ibid.*) argues that competition and diversification characterized the functioning of urban grain markets. The municipal exchange and granary, the Alhóndiga, was a central hub for the corn trade in Mexico City. City authorities tried to enforce the monopoly of the Alhóndiga. However, in practice, corn was sold in other venues, such as Indian *tianguis* (flea markets), the Church sale of the tithe, and *tiendas* (stores), and price controls were not binding (*ibid.*, pp. 95-98; Challú, 2013, p. 403). Furthermore, while local haciendas were the largest supplier to the Alhóndiga (and stood to obtain short-term gains) there were other producers supplying grain to the city (Challú, 2007, p. 77). Wheat was mostly sold as flour by mills in Mexico City. The wholesale exchange operated without many restrictions in contrast to retail, where bakers had to comply with price and quality of bread regulations (*ibid*, pp. 146-147).

1.2. Environment

We now describe and develop the model economy formally. There are two (representative) agents: a hacienda and a household. The hacienda grows maize and wheat and sells the crops in a local food market to the household. Time is discrete and continuous forever. Every period represents half a calendar year. Maize is planted in the early period and harvested in the late period (of the same calendar year). Wheat is planted in the late period and harvested in the early period of the subsequent calendar year. The index W indicates the period when wheat is harvested and M the period when maize is harvested. Note that the calendar year starts with the wheat harvesting season.

Two factors determine agricultural output. First, the hacienda chooses *how much land to use* to grow maize and wheat in the early and late periods, respectively. The usability of land varies. The hacienda employs land that requires little work first and extends production against ever-increasing marginal costs. Second, the seasonal climatic conditions, $F \in \{F_w, F_m\}$, vary randomly between periods and determine the *crop yield*. The seasonal climatic conditions and, in turn, the crop yield are determined after the land use decision and before the harvest occurs in the subsequent period. Hence, the hacienda commits the land before the crop yield is known and has to form expectations about the upcoming seasonal climatic conditions.

The hacienda can store maize and wheat for the subsequent period. Storing any crop is costless. However, wheat can only be stored from the early sub-period, when it is harvested, to the late sub-period; similarly, maize can only be stored from the late to the early sub-period of the subsequent period. Hence, each crop can only be stored for one period and current and expected future market prices determine the storage decision. Therefore, haciendas face two dynamic problems: *how much land to* *use* to grow one crop and harvest it in the next period, and *how much* of the other crop *to store* and sell in the next period.

The intra-period sequence of events goes as follows. The period's seasonal climatic condition F is given. The hacienda starts the period with a stored amount of the crop that was harvested last season and land committed to growing the other crop. All stored crop from the last period is sold. The committed land and the seasonal climatic conditions determine the harvest of the other crop. The hacienda decides how much of the recent harvest to sell and how much to store for the next period. Markets clear. Further, the hacienda commits land to grow the next season's crop. Finally, nature draws a random variable F which determines the seasonal climatic condition and crop yield for the subsequent period.

1.3. The household

The representative household derives utility from consuming maize and wheat, earns a fixed income in each period, and spends all income on food. Households have no storage and no other wealth-saving technology: the economic problem of the household is separable between periods.

The utility is given by $U = (\delta C_m^{\sigma} + (1-\delta)C_w^{\sigma})^{\frac{1}{\sigma}}$, i.e., the constant elasticity of substitution (CES) function, where C_i is the consumption quantity of good $i \in \{\mathbf{m}\text{aize},\mathbf{w}\text{heat}\}$. The (relative) weight of each good is determined by δ. Our parameter of interest, the elasticity of substitution, is denoted by ϵ and can be recovered from the CES utility function by $\epsilon=1/(1\text{-}\sigma)$. The budget constraint is $\sum_{i\in\{\text{m, w}\}} P_i C_i = \text{Income}$, where income is exogenously given.[14] Maize is the numéraire good, so P is the price of wheat in terms of maize. The optimality condition for the consumption decision of a household is the equality between the (relative) price and the marginal rate of substitution, or

$$P = \frac{1-\delta}{\delta} C^{1/\epsilon} \tag{1}$$

where C is the consumption ratio of maize over wheat, or $C = C_m / C_w$. The CES utility is a standard choice in the literature, and its parsimonious solution facilitates the work and debate.

The elasticity of substitution, $\epsilon = (\partial C/\partial P)(P/C)$, is constant across consumption levels and ratios and independent of δ. If the household's preference for maize increases then δ increases. An increase in δ depresses the ratio $(1-\delta) / \delta$ which, in turn, increases the maize-wheat consumption ratio C, holding the relative price constant.

1.4. The hacienda

The production function is given by $H_i = F_i L_i$ where $L_i \geq 0$ is the land assigned to grow crop $i \in \{m,w\}$. The yield of crop i, $F_i > 0$, varies over time according to the climatic conditions.[15] The climatic conditions F_i follow a Markov process: any realization in one period depends solely on the realization of the previous period. We describe this process in section 2.2.

As mentioned above, land use is costly. We capture these costs with the function $l_i(L_i)$ which depends on the crop i and is strictly increasing and strictly convex. Further, the hacienda can store each crop for one period. The hacienda has the following valuation in an early period when the wheat-harvesting occurs

$$V_w(S_m, L_w, F_w) = \max_{C_w, L_m} \left\{ C_m P_m + C_w P_w - l_m(L_m) + \beta \mathbb{E}\left[V_m(S_w, L_m, F_m) \right] \right\} \tag{2}$$

The hacienda enters the period holding S_m units of maize from the last (late) period where it also dedicated L_w units of land to the production of wheat. The third state variable F_w is the seasonal climatic conditions that determine the crop yield.

All the stored maize must be sold at price P_m as it cannot be stored further so that $S_m = C_m$ and the intra-period profit from selling the stored maize is $\pi_m = C_m P_m$. The harvest is $H_w = F_w L_w$ units of wheat. The wheat budget constraint, $H_w = C_w + S_w$, determines the use of the harvest. C_w is the amount of wheat brought to the market and sold at price P_w, yielding the intra-period profit $\pi_w = C_w P_w$. The amount of wheat stored to be sold in the next period is denoted S_w.

The last decision the hacienda has to make is to dedicate land L_m to the production of maize, which incurs costs $l_m(L_m)$. The future benefit of these two choices is captured by the (discounted) continuation value $V_m(S_w, L_m, F_m)$ which, in turn, depends on the unknown state F_m, the climatic condition that determines the maize yield. Hence, the hacienda forms expectations $\mathbb{E}[\cdot]$. Finally, the wheat storage S_w, the wheat sale C_w, and the land use for the maize production L_m are subject to non-negativity constraints.

The hacienda's valuation for the maize-harvesting period looks similar and reads

$$V_m(S_w, L_m, F_m) = \max_{C_m, L_w} \left\{ C_w P_w + C_m P_m - l_w(L_w) + \beta \mathbb{E}\left[V_w(S_m, L_w, F_w) \right] \right\} \tag{3}$$

The representative hacienda starts the late period holding S_w units of wheat from the harvest in the early period of the same calendar year, ready to be sold at price P_w. L_m of land is planted to grow maize and F_m are the climatic conditions. L_m and F_m jointly determine the maize harvests $H_m = F_m L_m$, which can either be sold (C_m) at price P_m or stored (S_m). The hacienda commits land to grow wheat, L_w, which incurs cost $l_w(L_w)$. The benefit of the land use (L_w) and maize storage (S_m) is reflected in the continuation valuation $V_w(S_m, L_w, F_w)$. Equations (2) and (3) represent the recursive representation of a contraction mapping problem which can be solved using standard techniques (e.g., Stokey, 1989).

1.5. Market clearing, specifications, and solutions

The markets for maize and wheat clear in equilibrium. Hence, an equilibrium exists, and this closes the model. To bring the model to the data we need to specify the cost func-

[14] Alternatively, households could supply labor for a wage which, in turn, determines the income. But the elasticity of substitution is independent of the income for a CES utility function. Hence, the level of income is irrelevant, and only relative scarcity is relevant for our households. We could also nest our model in a more complex environment where the consumption of maize and wheat are additive-separable from the rest of the consumption choices, and some fraction of the income is dedicated to the consumption of crops.

[15] Note that there is a one-to-one relationship between the climatic conditions and the crop yield; we use the terms interchangeably.

tion $l_i(L)$ and operationalize the process that describes F. We discuss the process describing the seasonal climatic condition F in the data section, which comes next.

We employ the following specification for the cost function

$$l_i(L) = \gamma_i \left(\frac{1}{1-L} - 1 \right), \; i \in \{m,w\} \tag{4}$$

to capture the effort of using land to grow crops. Notice that the marginal cost is zero when no land is used, $\partial l_i / \partial L \rightarrow 0$ as $L \rightarrow 0$, and increases to infinity when all land is used, $\partial l_i / \partial L \rightarrow \infty$ as $L \rightarrow 1$. This ensures that the optimal land use is between zero and one, $0 < L < 1$, for $\gamma_i > 0$.

We have six unknown parameters. The two preference parameters, δ and ϵ, are determined by our estimation. We set the half-yearly discount factor β to 0.995 so that the implied yearly discount rate is about 1%. We obtain the exogenously-given household income from Arroyo-Abad *et al.* (2012), who report that the daily wage for an unskilled construction worker in Mexico City was 3 reales and constant between 1732 and 1815. We thus set the income of a household at 3 reales in our estimation. Finally, the estimation determines the two cost parameters, γ_m and γ_w.

2. Data

We have several data needs to estimate the elasticity of substitution, ϵ. First, the planting dates of wheat and maize determine when haciendas allocate land to the respective crop production and, in turn, what haciendas know when they plant. The harvesting dates, on the other hand, determine which historical market price observations P led to market clearings. The suitability of the seasonal climatic condition index F for the agricultural production function is also influenced by the planting and harvesting dates.

Second, we measure F using biannual tree-ring growth measurements near Mexico City and discretize the state realizations using the approach brought forward by Tauchen (1986). This simplifies the analysis as we construct Markov transition matrices from the early to the late period and from the late period of one year to the early of the next. We use these discrete realizations as input to the model.

Third, historical price data allow us to estimate the model described in the previous section. By calibrating the parameters, the estimation ensures that the predicted price movements align with actual historical prices. As a result, the underlying preference and technology parameters can more accurately reflect the real world.

In addition, we utilize yearly volume data for maize and wheat imports into Mexico City from Florescano's (1969) and García Acosta's (1988), respectively. To ensure comparability, we convert the data into caloric units and utilize the wheat import volume data in the subsequent estimation.[16] We examine the reliability of the reported volume quantities in section 4.

We describe the sources and preparation of the historical price series in the next subsection. We also discuss the seasonal timing and our use of tree ring data to obtain a biannual climatic condition index F. We conclude with the discretization of the state space of F and derive a Markov transition matrix that we employ in our estimation. We briefly compare our estimated time-effects with the self-calibrating Palmer Drought Severity Index (scPDSI). We discuss the volume data together with the estimation results in the next section.

To keep the analysis brief, we discuss some data issues in more detail in the Appendix. Details on the preliminary statistical analysis of the tree rings can be found in Appendix B.1.b. A more detailed deliberation of the econometric results for the time-effects is presented in Appendix B.1.c. We provide details for the construction of the Markov transition matrix in B.1.d.

2.1. *Crop prices*

a. Data source

We use maize price data reported by Florescano (1969) and wheat price data from García Acosta (1988). The maize data is taken from the official accounting books of the Pósito and Alhóndiga of Mexico City *(libros de cuentas)*. Maize that entered the city had to be first taken to the Alhóndiga and the official sale of maize was confined to its patios. Similar to maize, wheat that entered the city had to be declared. García Acosta (*ibid.*) obtained the wheat price data from transaction records declared to the *Tribunal de la Fiel Ejecutoria* by wheat buyers in Mexico City. Bakers bought wheat flour from the mills in the city.

The volume of maize imported to Mexico City and reported in Florescano (1969) suggests that less than 300 calories per day and inhabitant were sold through the Alhóndiga. The corresponding quantity for wheat reported in García Acosta (1988) is about 1,600 calories. These quantities are significant when considering the daily requirement of consuming 1,940 calories (Arroyo-Abad *et al.*, 2012). However, it is important to note that this rough calculation does not account for the animal feed. Therefore, these quantities are unlikely to represent the total amount consumed in Mexico City, and the maize quantity seems particularly low. Nevertheless, the prices offered in official channels were subject to competition in informal trading channels, and they probably reflected the general price level in Mexico City.

We have access to price data for both crops, maize and wheat, from 1741 to 1812. The prices are specific to each day, but the frequency of reporting is low, with more than 87% of wheat prices and over 90% of maize prices being missing. Additionally, the reporting pattern changed over the years, with some years having nearly daily observations and others having only a few observations. Despite this, we do not suspect that the missing observations indicate a non-random selection process.

b. Data summary

We normalize the measuring units to 1,000 calories (rather than a weight measure) using data from Arroyo-Abad *et al.*, (2012). Hence, non-caloric factors, such as differences in taste, must explain the price differences. Figure 1, top panel, displays the normalized price observations for wheat and maize.

[16] In particular, one carga of wheat translates to about 185 kg and 1 kg of wheat has 3,500 calories. Similarly, one fanega of maize translates into 51.3 kg and 1 kg of maize has about 3,840 calories.

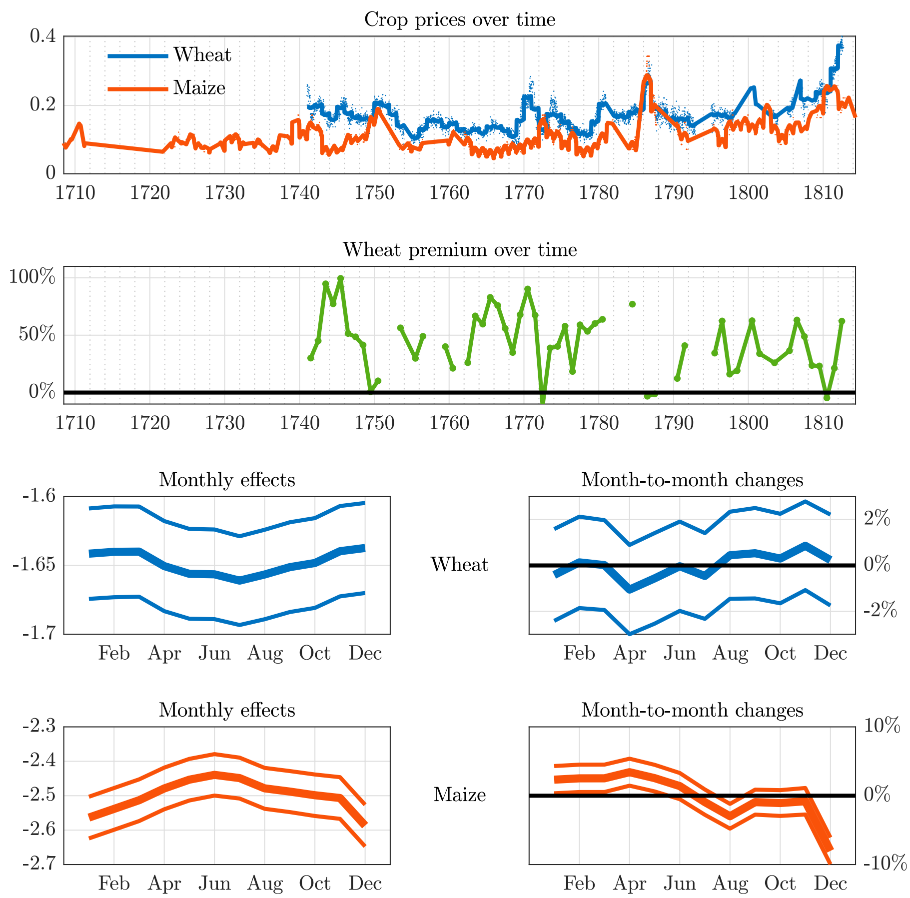

Figure 1. Crop prices (1,000 calories per real).

Notes: The crop price units are normalized to 1000 calories. Top panel: price observations for wheat (blue dots) and maize (red dots) per 1000 calories. The lines use predictions from a regression of prices on yearly and monthly dummies. Second panel: differences between the yearly predictions for wheat and maize so that a positive value represents the wheat premium. The bottom half presents the within-year variation of log-prices. The two left panels of the bottom half show the monthly coefficients with confidence intervals that are two standard deviations from the mean. The right panels show the month-to-month changes in these coefficients derived from the Delta method with similar confidence intervals. The top (bottom) two panels of the bottom half show the results for wheat (maize).

Data sources: García Acosta (1988) and Florescano (1969) for wheat and maize prices, respectively.

There are four noteworthy statistical regularities. First, there is a premium attached to wheat over maize which implies that wheat and maize are not perfect substitutes. The average and the median calorie-denominated price for wheat are above the respective price statistics for maize (Table 1). The difference in the average and median prices suggests that wheat was about 37% and 55% pricier, respectively. Further, the existence of a wheat premium is visually supported by the top panel of Figure 1. Calorie-denominated wheat prices are, on average, above the maize prices. Finally, we run a regression with separate yearly and monthly dummies for wheat and maize log-prices, and subtract the sum of the yearly dummies of the maize prices from the sum of the yearly dummies of the wheat prices. The resulting Wald-statistic is 6595 —which is ξ^2— distributed with one degree of freedom. We reject the null hypothesis that the wheat premium is zero by any conventional level of significance.[17] The second panel of Figure 1 shows the difference between the yearly coefficients, i.e., the percentage difference between maize and wheat. In other words, it shows the wheat premium over time. While there are some cyclical patterns, we do not detect a trend in this premium.

Second, there is considerable cyclical and co-movement between the prices (Figure 1, top panel). The statistically significant correlation coefficient for the days for which we have both prices available is 0.64. This (imperfect) correlation suggests that wheat and maize are (at least) not perfect complements. However, this may also be attributable to variation in production.

Third, both prices are subject to mild inflationary pressure. We regress log-prices of wheat and maize separately on a constant and a time trend (in years). Both price series increased significantly, however on a moderate scale. Wheat prices increased by about 0.5% per year, while maize prices increased by about 1.1% per year.

Lastly, to the extent that the expenditure on food represents a large fraction of household spending, crop prices can be related to nutritional and economic welfare. Arroyo-Abad *et al.* (2012) show that the daily wage stagnates for most of our period, at 3 reales for every year between 1732 and 1815. Hence, crop prices are indicative of real income, and the two series show how real income varied over time.

[17] Wheat was also somewhat more costly to produce since it required more inputs (irrigation and extra labor), and more capital outlays (Challú 2007, p. 159).

Table 1.
Summary statistics for price data and the agricultural calendar

	Wheat	Maize
First observation	03-Jan-1741	30-Jun-1708
Last observation	06-Aug-1812	30-Apr-1814
Span in days	26,148	38,655
Number of observations	3,363	3,690
Lowest price	0.082	0.043
Highest price	0.403	0.344
Mean price	0.175	0.128
Median price	0.166	0.107
Standard deviation price	0.049	0.065
Planting	October	April
Harvest	April	December

Notes: The crop price units are normalized to 1000 calories. The planting and harvesting dates are from Gibson (1967, p. 339).

c. Agricultural calendar

The lower half of Figure 1 shows the monthly coefficients of the above-mentioned regression of wheat (top left panel in the lower half) and maize log-prices (bottom left panel in the lower half) on yearly and monthly dummies. A strong decrease in the cyclical monthly pattern of crop prices would imply the market is flooded with a crop after harvest. The right panels in the lower half of Figure 1 show the estimated month-on-month changes derived using the Delta method. Indeed, in line with Gibson (1967, p. 339), we find the steepest month-on-month changes for wheat prices in April and for maize prices in December. Hence, we choose April as the harvesting date for winter wheat and December as the harvesting date for maize.

The calendar implies that haciendas use the seasonal climatic conditions of the first half-year, when maize is planted (in April), to determine the growth of winter wheat; in turn, the seasonal climatic conditions of the second half-year determine maize growth.

d. Price data for market clearing

We calculate the median caloric wheat and maize price for April and December for each year between 1721 and 1814. This leaves us with 188 half-yearly observations. We observe both prices in 67 periods and no prices in 39. We observe only wheat prices in 52 periods and only maize prices in 30. As noted, we have no reason to believe that these data omissions are subject to a non-random selection process.

2.2. Climatic conditions

a. Data source

The goal of the next analysis is to elicit a seasonal climatic condition index *F* that governs the crop yield. It is critical that this seasonal climatic condition index *F* correlates with conditions that are favorable to crop production.

Tree ring measurements can provide useful proxy information about the local climate where a tree is located. The US National Oceanic and Atmospheric Administration's (NOAA) National Centers for Environmental Information maintain data sets that have an unbalanced panel data structure, with repeated observations for the same cross-sections (or "trees"). These trees are all of a single type, and the area covered by the data sets is relatively small. Due to these characteristics, we refer to these data sets as "forests". Twenty such forests, located in an area that is now part of Mexico, contain biannual measurements of tree ring growth.

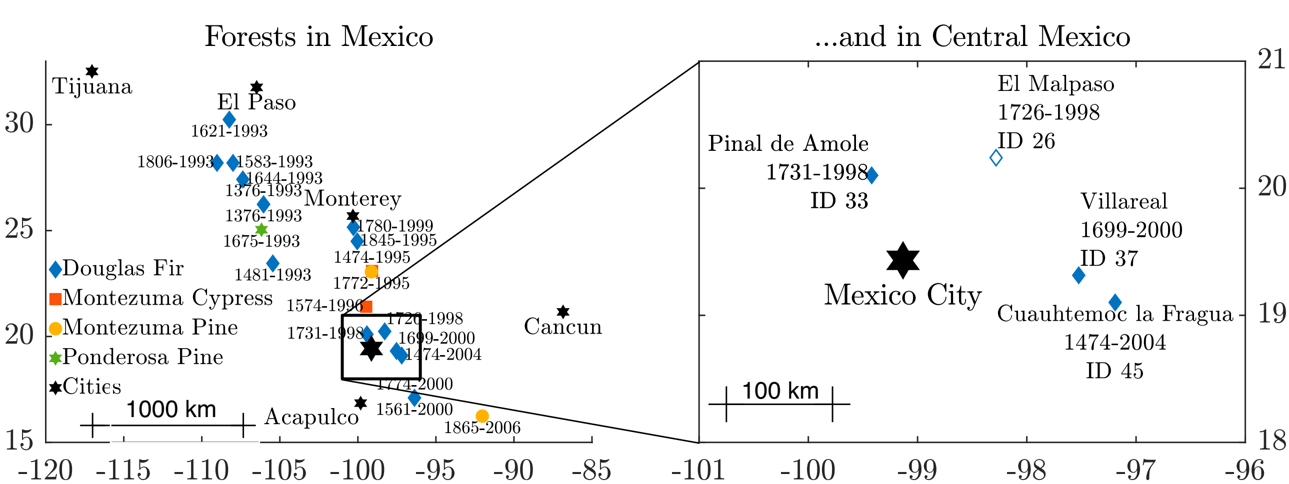

Figure 2. Tree ring data sets ("forests") in Mexico.

Notes: The left panel shows all forests in Mexico with biannual measurements, indicating the type of tree and the years for which the data is available. Five Mexican cities serve as point of reference. The square around Mexico City is the area of interest which is enlarged in the right panel.

Figure 2 left panel illustrates the locations of these forests relative to several notable Mexican locations, while the right panel shows the region around Mexico City where our price data was collected. Of the twenty forests, four are located near Mexico City. To standardize the tree ring growth measurements, we apply the logarithmic function. "Early wood growth" refers to tree growth during the first six months of the year, while "late wood growth" refers to growth in the last six months. We denote early (late) wood log growth of tree $i \in \mathbb{I}$ in year $y \in [1474, 2004]$ by $e_{i,y}$ ($l_{i,y}$) where $|\mathbb{I}|=383$ denotes the total number of trees. We also assign each tree to a forest so that $i \in \mathbb{F} = \{26,33,37,45\}$ where the numbers stem from the NOAA identifier.

b. Estimation

Our objective is to construct a biannual climatic condition index that is specific to each period. This index must be based solely on the concurrent climatic conditions. In econometric terms, we seek to identify time-effects that are independent of past climatic conditions; past conditions should not influence current tree-ring growth measurements.

To obtain proxies for the concurrent climatic condition F, we use a panel data model to estimate time-effects.[18] The growth pattern of tree rings is, in part, determined by constant, tree-specific fixed-effects: using tree-specific dummies explains about 22% and 27% of the total variation in the early- and late-wood growth, respectively. Further, a period-specific mean (of all tree ring growth observations) is likely subject to an attrition bias because the panel is not balanced: frail young trees survive a spell of some good years, grow a lot in their early years, and enter the point of data collection. This amounts to a selection issue.[19] Further, an initial analysis suggests that the growth process has an auto-regressive component. (Appendix B.1.c discusses possible explanations.) The dynamic panel data regression model becomes

$$g_{i,t} = \overbrace{\beta \times age_{i,t} \times (1 - l_{i,t}) + \sum_{q=1}^{Q} \psi_q \times age_{i,t} \times l_{i,t}}^{\text{age profile}} + \tag{5}$$
$$+ \underbrace{\sum_{\hat{t}=\underline{t}}^{\bar{t}} \delta_{\hat{t}} \times \mathbb{D}[\hat{t} = t]}_{\text{time-effects}} + \underbrace{\sum_{v=1}^{V} \omega_v \times g_{i,t-v}}_{\text{auto-regressive component}} + \mu_i + \epsilon_{i,t}$$

where $g_{i,t}$ is the log growth of tree i in period t, and the variable $l_{i,t}$ takes the value of "1" if the period t is "late", and "0" otherwise.

The first line of equation (5) displays the age profile. Young trees grow faster. More importantly, a spell of some good years leads to more observations, as young trees survive a frail period in their lives, and also to more high-growth observations. This biases a naive period-specific mean upwards after a spell of some good years. Our remedy is to include a linear age trend for early-wood growth so that trees grow, on average, β in every early period. The Bayesian Information Criterion (BIC) decides the polynomial order Q for late-wood growth.[20]

The second line starts with the time effects, captured by $\delta_{\hat{t}}$, which serve as climatic condition index in our main estimation below. The indicator function $\mathbb{D}[A]$ is equal to one if statement A is true, and zero otherwise. The remaining part of the second line contains the auto-regressive component and the tree-specific and idiosyncratic effects. The Bayesian information criterion also determines V, the auto-regressive order. The parameter μ_i is the tree-specific effect on growth and $\epsilon_{i,t}$ captures the idiosyncratic noise.

The preferred specification is reported in the last column of Table 7 in the Appendix.[21] It employs $V=2$ auto-regressive lags and a linear late-wood age model ($Q=1$). We do not report the estimated coefficients here. The R^2_{within} from a regression without the auto-regressive component ($V=0$) grows from 0.31 to 0.53 (with auto-regressive component), and the $R^2_{overall}$ grows from 0.53 (without auto-regressive component) to 0.67 (with auto-regressive component). The Bayesian Information Criterion also favors the dynamic specification.

c. Time-effects vs the Palmer Drought Severity Index

How well do the estimated time-effects reflect climatic conditions? In order to answer this question we compare the time-effects with a related measure, the self-calibrating Palmer Drought Severity Index (scPDSI), as presented in the Mexican Drought Atlas (MDXA, compare Stahle *et al.,* 2016). The scPDSI focuses on soil moisture which is not directly measurable for our period of investigation (compare Wells *et al.*, 2004). The scPDSI is based on modern measurements of soil moisture levels and extrapolates these values into the past using tree ring measurements. Rather than using the scPDSI for our study, we employ tree ring measurements directly without loosing information through an extrapolation process.

The early (late) time-effects $\delta_{e,t}$ ($\delta_{l,t}$) correspond to the coefficients $\delta_{\hat{t}}$ when $\hat{t}$ refers to the early (late) period of year t. The red line in Figure 3 shows the normalized early and late time-effects, i.e., the re-centered time-effects are divided by their respective standard deviation. The blue line in Figure 3 shows the scPDSI, for the location of Mexico City.

While they are not perfectly correlated, their correlation coefficient is 0.6 and significantly different from zero. A notable event that occurred in our estimation window is the *el año del hambre* in 1785-1786. Our time-effects pick up on unfavorable climatic conditions, although it is not the worst time-effect. The lowest single index value can be found in the early 1730s.

[18] For the full preliminary analysis of the tree ring data, see Appendix B.1.b. Notably, we drop the "El Malpaso" forest ($i = 26$) to the Northeast of Mexico City as its growth is uncorrelated or detrimental to the growth of the remaining three forests (Fig. 2, right panel). Dropping this forest implies that $|\mathbb{I}|=333$ and $\mathbb{F} = \{33,37,45\}$.

[19] That is, the time-effects and the tree-specific fixed-effects are correlated which invalidates the random-effects assumption where the tree-specific fixed-effects can be left in the residual of the regression. The period-specific mean is similarly biased. Luckily, the number of observations is very large so we do not need to rely on the random-effects assumption. Rather, we only employ the fixed-effects assumption.

[20] This solution mirrors the use of an age-dependent cubic smoothing spline in the MDXA (Stahle *et al.*, 2016).

[21] See the Appendix for an explanation of all possible perturbations.

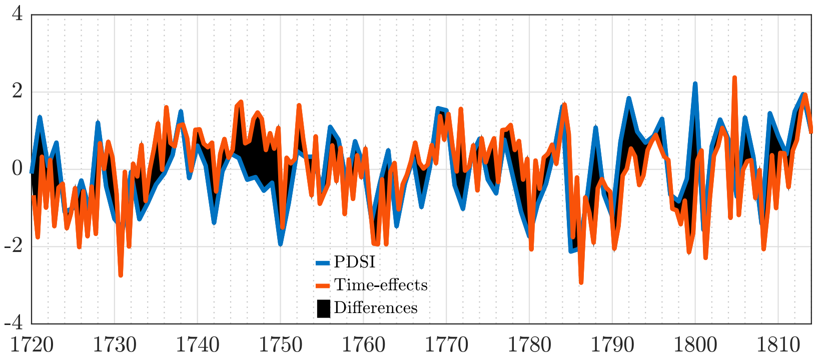

Figure 3. Estimated time-effects vs. Mexican Drought Atlas (MDXA) for Mexico City.

Notes: Comparison of estimated time effects with a self-calibrating Palmer Drought Severity Index (scPDSI), as constructed in the Mexican Drought Atlas (MDXA) by Stahle *et al.*, (2016).

d. Markov states and transition matrix

We transform the estimated time-effects into the climatic conditions used in the economic model. We discuss the procedure in detail in Appendix B.1.d and summarize here the main steps. First, we define the early (late) climatic condition F_w (F_m) in year t which determines the wheat (maize) yield as the time-effects δ_t. Next, we discretize F by grouping realizations, as proposed by Tauchen (1986). We divide F_w and F_m climatic conditions into groups using the k-means algorithm. The silhouette method determines the optimal number of clusters with a range from 4 to 8. The optimal choices are 4 and 5 for the climatic conditions that determine the wheat and maize yield, respectively. Hence, the early period has the possible state realizations $W = \{W_1, W_2, W_3, W_4\}$ and the late period has the state realizations $M = \{M_1, M_2, M_3, M_4, M_5\}$. See the top panel of Figure 8 in the Appendix.

Finally, we create two Markov transition matrices. A Markov transition matrix describes the probabilities of transitioning from any state in one period to any other state in the subsequent period. In particular, the Markov transition matrix $\prod_{W2M}$ (with elements π_{WM}) describes the probabilities of transitioning from any state in the early period (W) to any state in the late period (M). Similarly, the transition matrix describing the transition probability of going from state M to W is denoted $\prod_{M2W}$ (with elements π_{MW}).[22]

This discretization allows us to rewrite equations (2) and (3) as

$$V_M(S_w, L_m) = \max_{C_m, L_w} \left\{ C_w P_w + C_m P_m - l_w(L_w) + \sum_{E=1}^{4} \pi_{M,W} \beta V_w(S_m, L_w) \right\} \quad (6)$$

$$V_W(S_m, L_w) = \max_{C_w, L_m} \left\{ C_m P_m + C_w P_w - l_m(L_m) + \sum_{L=1}^{5} \pi_{M,W} \beta V_M(S_w, L_m) \right\} \quad (7)$$

where W and M are generic states that determine the wheat and harvest yield, respectively.

The level of the realizations that describe the crop yield cannot be negative. This introduces four more parameters to our estimation below. In particular, we estimate the lower states $\widehat{W}$ and $\widehat{M}$ and the range of the states ΔW and ΔM so that

$F_w = \widehat{W} + \Delta W\,(W_k - W_1) \geq 0$ in state W_k $\forall k \in [1,2,3,4]$ and

$F_m = \widehat{M} + \Delta M\,(M_k - M_1) \geq 0$ in state M_k $\forall k \in [1,2,3,4]$

[22] Table 8 in the Appendix provides the numerical values.

3. Estimation results

3.1. Moment selection

We carefully choose the moments used in the estimation of the model to aid in the identification of specific parameters. The levels of crop prices are linked to the cost parameters (γ_m and γ_w) and the level of agricultural production ($\widehat{W}$ and $\widehat{M}$). The dispersion of crop prices provides information about the range of climatic conditions (ΔW and ΔM). We use the co-movement of prices to identify the relative price response and thus the elasticity of substitution ϵ, which is the primary parameter of interest. The wheat premium, or the price difference between wheat and maize, reflects the relative weight households place on the consumption of maize and hence informs the estimation of the parameter δ. It is important to note that these connections are only at a first-order level, and there may be other, more nuanced links between the moments and the parameters being estimated. Finally, we use the wheat volume data of city imports from García Acosta (1988) to normalize the wheat prediction. The estimation works quite well without using the wheat volume data, and the parameters only change slightly. However, it allows us to contrast the maize consumption with reported data.

We thus have thirteen moments to estimate eight parameters.[23] The estimations below are based on the principles of the Generalized Methods of Moments (GMM) and the Empirical Likelihood (EL). We employ 1-step and 2-step GMM estimation where the latter down-weights moments that are inherently more dispersed. To compute the moments, we take the logarithm of the price and volume data and discard model predictions whose corresponding empirical observations are missing.

3.2. Statistical fit

Table 2 summarizes the results. To evaluate the model fit in a more intuitive way, we define the loss as the sum of the squared average differences between the model's predictions and the observed data. All estimations provide a good fit for the data. The loss function values hover around 0.002 which is small in our view. The level of predicted prices corresponds to

[23] Table 2 contains the list of 13 moment conditions and shows the 8 parameters.

the empirical prices quite well. The largest deviation from the empirical prices occurs in the 1-step GMM estimation for wheat prices in the early period. The predicted prices are about 4% lower. The variances of the predicted prices are larger than their empirical counterpart. In particular, the dispersion of empirical wheat prices is much smaller compared to our model predictions. This predicted over-dispersion translates into the over-prediction in the co-movement of wheat and maize prices. The moment conditions of the average wheat premium are matched quite well as these moments relate to the price levels. Finally, the model predicts an average maize harvest that fits the historical city import measurements well (Figure 5).

Table 2.

Targeted data moments, model predictions, summary statistics, and model parameters from the main estimation

				Model predictions		
			Data	**1-step GMM**	**2-step GMM**	**EL**
Average Price	Maize	Early	-2.233	-2.237	-2.250	-2.249
		Late	-2.299	-2.300	-2.291	-2.295
	Wheat	Early	-1.786	-1.743	-1.756	-1.754
		Late	-1.801	-1.791	-1.781	-1.784
Price Variance	Maize	Early	0.172	0.235	0.206	0.190
		Late	0.126	0.131	0.132	0.129
	Wheat	Early	0.060	0.228	0.201	0.185
		Late	0.069	0.133	0.133	0.130
Price Covariance		Early	0.082	0.232	0.203	0.188
		Late	0.077	0.132	0.133	0.130
Wheat Premium		Early	0.462	0.493	0.494	0.494
		Late	0.522	0.509	0.510	0.510
Yearly Wheat Consumption			0.457	0.469	0.458	0.453
				Statistics		
loss				0.002	0.002	0.002
Overidentification statistic				0.023	1.878	1.981
p-value				1.000	0.866	0.982
				Parameters		
δ				0.7513*** (0.1724)	0.7516*** (0.1813)	0.7438*** (0.0080)
ϵ				2.243* (1.275)	2.246* (1.359)	2.307*** (0.0599)
γ_w				2.047*** (0.7148)	2.024*** (0.2098)	2.031*** (0.4635)
γ_m				0.1205*** (0.0167)	0.1196*** (0.0101)	0.1186*** (0.0117)
$\widehat{W}$				3.162*** (0.1647)	3.122*** (0.1089)	3.104*** (0.1463)
$\widehat{M}$				87.47*** (19.07)	87.63*** (7.788)	87.49*** (12.88)
ΔW				0.0050 (0.2108)	0.000*** (0.000)	0.0050 (0.1938)
ΔM				29.69*** (10.83)	29.19*** (5.351)	29.30*** (7.217)

Notes: The first column contains targeted data moments. The second and third column contains the predicted moments and statistical results for the 1-step and 2-step GMM estimation, respectively. The last column is based on the EL estimation. See the text for variables and data sources; p-value *** p<0.01, ** p<0.05 and * p<0.1.

Since we have 5 degrees of freedom, we are able to determine whether the moment conditions are over-identifying the model. Over-identification can be detected when the estimation uses more moment conditions than parameters.[24] Both, GMM and EL provide an identification test based on the loss value of the respective objective functions. However, all tests cannot reject the null hypothesis that the model is over-identified. We conclude that our moments do not contradict themselves through the eyes of the model and we do not reject the model itself.

The corresponding parameter estimates are quite close to each other among the three different estimations. The estimates for δ hover around 0.75 which implies a strong preference for maize over wheat. This might come as a surprise because the empirical price levels suggest that households were willing to pay a premium for wheat. However, the production parameters suggest that maize was cheaper to plant, with γ_m estimates around 0.12 while the γ_w estimates are all larger than 2. This is in line with Challú (2007, p. 159) who states that wheat was more costly to produce since it required more inputs (irrigation and extra labor), and more capital outlays. Further, the maize crop yield was much larger than the wheat crop yield. The lower state estimates for $\widehat{M}$ are (at least) 27-fold of the $\widehat{W}$ estimates. This implies that colonial Mexicans planted far more maize than wheat and this relative abundance of maize, rather than consumer preferences, led to the wheat premium.

Finally, the estimates for ϵ are all around 2.3. A 1% increase in the wheat-maize price ratio led to a 2.3% increase in the maize-wheat consumption ratio. An elasticity of substitution above one implies that the quantity demanded of one crop increases as the price of the other crop increases, indicating that the goods are substitutes. In other words, households switch between the two crops, reducing their consumption of the more expensive crop when its price rises relative to the other.

To illustrate the relationship between the elasticity of substitution and the substitutability of crops, let us assume that the price of wheat increases while the price of maize remains constant. As a result, the per-unit expenditure on wheat will increase due to the price hike, while the demand for wheat will decrease, resulting in a decrease in expenditure. The net impact depends on the elasticity of substitution. In the case of a CES-utility-based expenditure function, the sign of the derivative of an individual crop's expenditure function with respect to its own price is determined by the value of ϵ. When ϵ is greater than one, the expenditure decreases, whereas when ϵ is less than one, the expenditure increases. Conversely, the relationship between the expenditure function and the cross-price is inverse. Therefore, with an elasticity of substitution of approximately 2.3, an increase in the price of wheat results in an increase in maize expenditure: households substitute wheat for maize as wheat becomes more expensive.

4. Living standards and the price sensitivity

4.1. Implications for welfare

To analyze the welfare implications of our estimates for late colonial Mexico, we calculate crop expenditure and inflation as indicators of well-being and compare them across policies. Let us return to our example of the family of four that requires 7,760 calories daily, as stipulated by Arroyo-Abad *et al.*, (2012). We continue to assume the household has CES preferences and the household income is fixed at 3 reales. The top panel of Figure 4 plots the fraction of the average income spent on crops when our household follows a fixed basket ($\epsilon \to 0$), a "cheap" basket ($\epsilon \to \infty$), and the estimated policies from the EL estimation. In order to provide a clearer picture, we smoothened the series with a 365-day filter.

Crop expenditure was a substantial part of household spending in colonial Mexico. Even the "cheap" basket spent about 29.2% on average of income on wheat and maize (see Appendix Table 5). And the fixed basket, arguably the most expensive expenditure policy, consumed 36.3% on average of income. The fraction of income spent on crops for our estimated policies hovers around 29.8. The most notable year of nutritional distress is *el año del hambre* in 1785-1786 when even the "cheap" basket policy consumed 84.8% of the income of a laborer. (Note that this extreme observation derives from the non-smoothened daily calculation, which is not pictured in Figure 4). Unsurprisingly, the time trend and the cyclical patterns of all expenditure policies correlate strongly with the original price data (Figure 1).

The trends in the crop expenditure estimations are in line with other studies using real wages as indicators of past welfare. The sharp increase and fall around 1751 is corroborated in the estimations based on fixed-baskets for Mexico in Arroyo-Abad *et al.* (2012, Fig. 2) and Challú and Gómez-Galvarriato (2015, Fig. 1), as is the sharp increase in crop expenditures (downfall in real wages) around 1786. The overall reduction in welfare in the last decades of the 18th century is also observable, albeit to a smaller degree, in the fraction of income spent on crops.

Inflation, which refers to a sustained rise in consumer prices, is also a common macroeconomic indicator of well-being that highlights changes in the cost of living over time. Inflation is a especially relevant measure of the perception of living standards of day laborers because their nominal wages remained relatively fixed during the time period of our study. On average, our measure of inflation is within moderate levels, ranging from 1.4 (using the fixed basket) to 2.8 (using the cheapest basket) per year (Figure 4). These ranges are in accord with the price indexes in Challú and Gómez-Galvarriato (*ibid.*, p. 97) and other price indexes, with an average annual inflation rate of 2 per cent from 1775 to 1809. The sustained rise in inflation indicates a sustained fall in laborers' well-being during the period.

[24] The simplest example to motivate over-identification is the estimation of a mean parameter μ of a variable x using two instruments, y and z. There are two moment conditions, $\mathbb{E}[(x\text{-}\mu)\,y]$ and $\mathbb{E}[(x\text{-}\mu)\,z]$. The model is over-identified if $\mu_y \neq \mu_z$ in a statistical sense.

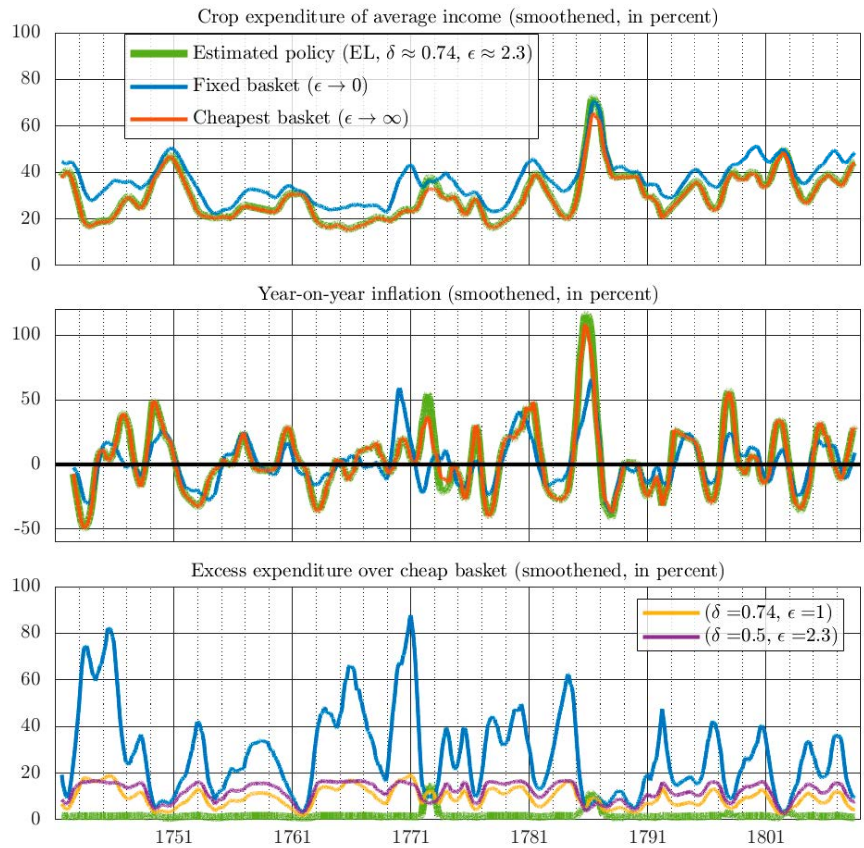

Figure 4. Expenditure and inflation for a 7,760-calorie basket.

Notes: Expenditure as a fraction of average income (%) and year-on-year inflation for a wheat and maize consumption basket that requires 7,760 calories daily. The expenditure policies are based on a CES utility function as described in the text, whose elasticity of substitution, ϵ, is set to 0 (fixed basket), ∞ (cheapest basket), and the estimated parameters from the EL estimation. All lines are smoothed over 365 days. The top panel shows the expenditure for wheat and maize. The central panel shows year-on-year inflation for all policies. The bottom panel subtracts the cheapest basket from the fixed basket and the policies using EL-estimated δ and ϵ. The yellow and purple lines display two counterfactual policies.

The policies analyzed in our estimations exhibit an inflation rate of about 3% (Figure 4, central panel). This is mainly due to the fact that maize prices were consistently cheaper for most days, accounting for approximately 94% of our sample, while wheat prices were more volatile. Therefore, the cheapest basket was more sensitive to increases in maize prices and experienced higher inflation rates. The additional inflation observed for the estimated policies compared to the "cheap"-basket counterpart is a result of households gradually switching to more costly wheat consumption as it became relatively less expensive.[25]

The volatility of inflation across different consumption baskets is another crucial aspect of welfare. Household economic planning can be disrupted if the expenditure on staple food items experiences significant fluctuations. Our findings reveal that the estimated policies exhibit the highest volatility of inflation, with a standard deviation of 25.6%, while the fixed basket has the lowest at 17.1%. The cheapest basket falls in between these two extremes. This result is not surprising given that the cheapest basket mostly consists of maize, which is the more volatile price series. The fixed basket is more diversified, resulting in a more moderate level of expenditure volatility. Our estimated policies add to the volatility by shifting the consumption weight towards the relatively cheaper crop compared to the previous period.

The price data alone, however, do not provide information on excess expenditure, which we define as the percentage difference between an expenditure policy and the "cheap" basket (Figure 4, bottom panel). Excess expenditure represents the extent to which households can afford to enjoy a consumption basket that is not solely focused on caloric intake. For instance, a household employing the fixed basket policy spent an average of 29.1% more than the "cheap" basket. The average excess expenditure for our estimated policies is approximately 1.9%. The excess expenditure is largely influenced by the wheat premium, which is the difference between the prices of wheat and maize. As the wheat premium decreases, the excess expenditure of the fixed basket policy moves closer to that of the other two policies.

4.2. *Sensitivity of parameters*

How sensitive are our predictions with respect to our estimated parameters? We change both preference parameters, δ and ϵ, and recalculate the excess expenditure. The yellow line in the lower panel of Figure 4 shows the excess expenditure for a CES-household when $\delta \approx 0.74$, as stipulated by the EL estimation, and we lower ϵ from 2.3 to 1. Hence, households become less price sensitive and maize and wheat become less substitutable. However, they retain their relative preference for maize. Further, the purple line reflects the excess expenditure when we leave the elasticity of substitution at the estimated level, $\epsilon \approx 2.3$, but lower the relative preference for maize to $\delta = 0.5$. The excess expenditure increases from 1.9% with the estimated parameters ($\delta \approx 0.74$, $\epsilon \approx 2.3$) to about 9.6% for both modifications.

[25] The inflation of a consumption basket is essentially the weighted average of random variables. While the weight is fixed in the fixed basket and (mostly) fixed for the "cheap"-basket policy, the weight switches from maize towards wheat in the estimated policies as the wheat inflation was lower than the maize inflation for our window of investigation. See our discussion in section 2.1.

Thus, if our estimation had yielded an estimated elasticity of substitution closer to 1, all else constant, the estimated policy would have shifted closer to the middle ground between the two other policies, in terms of the different proxies for living standards in Figure 4. We would observe a similar shift if instead the relative weight that households put on maize were 0.5 (giving the same weight to maize and wheat), all else constant.

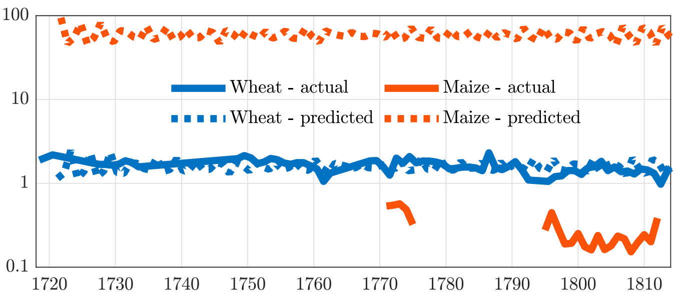

Figure 5. Volume data.

Notes: Wheat and maize import quantities, as reported by García Acosta (1988) Florescano (1969), respectively, and whole-year consumption levels of wheat and maize from model predictions. The vertical axis is in log-scale for a better visualization.

4.3. The (issue with) reported volume data

Lastly, we compare the quantitative predictions for maize consumption of our estimates with empirical volume data which was not used in the estimation. Typically, the purpose of such an endeavor is to establish external validity by utilizing data that was not utilized in the primary estimation. Nevertheless, we do the comparison to demonstrate that the volume data does not align with our estimations. Therefore, incorporating all of the available volume data would be pointless.

The estimations in the previous section employ and match the wheat imports into Mexico City quite well. However, the solid lines in Figure 5 reflect the yearly volume data for maize and wheat imports into Mexico City, as reported by Florescano (1969) and García Acosta (1988). Note that the vertical axis is log-scaled. The dashed lines show the predicted harvested quantities from the EL estimation.[26] While the wheat quantities match quite well, the predicted maize quantities overshoot the reported maize quantities by a manifold. The reported maize-wheat import ratio is 0.2. However, the predicted ratio is closer to 37, regardless of the estimation.

The reported numbers seem at odds with the conventional wisdom that maize was the dominant crop in the consumption portfolio of Mexican households at the time. In fact, the reported city import quantities of maize in Mexico City never exceeded 30% in terms of total calories obtained from maize and wheat. We suspect that the quantity of reported maize is too low to represent the consumption quantity. This is in line with observations by Challú (2007) and others that the Alhóndiga under-reported maize exchanges since maize was also exchanged at other venues in the city (as discussed in section 1), and grown for self-consumption.

[26] The other two estimations are quite similar in the level predictions.

5. Conclusion

Measuring the standards of living or quality of life during periods with limited documentation is a challenging undertaking. To overcome this challenge, some scholars have employed fixed consumption baskets to compute a welfare ratio that represents the cost of a subsistence package relative to the nominal wage. This ratio uses fixed quantities of consumption items to ensure households maintain the necessary caloric intake. While the prices of items in the basket may fluctuate, the quantity of items chosen remains constant, resulting in variations in the measure due to price changes rather than quantity choices.

Another method involves using the cheapest consumption basket, where individuals in a population meet their nutritional requirements by purchasing the crop at the lowest available price. While this approach may capture the spending habits of the poorest members of society, it overlooks the fact that people also consume more expensive food items. However, subsistence living requires innovation and adaptation as a response to necessity. Consumers naturally adjust their consumption choices based on the given prices.

We propose an alternate measure that captures the relative sensitivity to prices, which is conceptualized by the elasticity of substitution. This measure is not intended to replace existing methods but rather to demonstrate that households adapt to the scarcity of goods and resulting price changes, optimizing their consumption patterns. Therefore, we consider it a complementary approach. In fact, the argumentative fallacy that "the truth is always in the middle" is not necessarily incorrect in this case. Our analysis indicates that the estimated consumption expenditure lies between that of the cheapest basket and the fixed basket, although it is closer to the former. Our estimation suggests that the representative household assigns a higher value to maize than wheat (0.74). If households were to place equal weight on wheat and maize (0.5), our calculations would reveal an expenditure closer to the middle between the "cheap" and fixed baskets.

To the best of our knowledge, this is the first study that employs tree-ring measurements directly to feed an economic model calibration. Our estimates for the elasticity of substitution, which reflect price sensitivity, are based on changes in relative prices due to seasonal climatic conditions. These conditions are determined by tree-ring measurements, which are available at a bi-annual frequency, the minimum required for our estimation approach. Tree-ring measurements of this frequency are widely accessible. Care is needed in retrieving a climatic condition index and there may be discrepancies in the application of statistical methods in dendrochronological studies and mainstream economic applications. To minimize the potential for discrepancies, we cross-verify our index with other available studies.

The second crucial data source, prices, is also widely available for future studies. Like other approaches, we face limitations on the information on the quantities of goods available and consumed during the period. However, it is improbable that we have reliable quantity observations for staple food items for the majority of historical periods we wish to analyze.

This is the first study that employs the concept of an optimizing consumer to late-colonial Mexico, to our best knowledge. Our findings support a decline in living standards during this period, consistent with other research, and our analysis underscores the significance of accounting for taste preferences and market prices when evaluating historical living standards. We demonstrate that economic strain can encourage innovative consumption choices.

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APPENDIX

A. Expenditure and inflation

Table 5.

Fraction of available income spent on food, implied inflation for these consumption policies, and excess expenditure over the “cheap” basket over different time periods

Measure	Period	Cheap basket	Fixed basket	1-step GMM	2-step GMM	EL
Fraction of income	All	29.2	36.4	29.8	29.8	29.8
	1740s	29.2	37.9	29.6	29.6	29.6
	1750s	25.4	31.3	25.8	25.8	25.8
	1760s	20.4	28	20.7	20.7	20.7
	1770s	25	32.8	25.9	25.9	25.9
	1780s	37.6	43.4	38.9	38.9	38.9
	1790s	31.2	38.7	31.7	31.7	31.7
	1800s	37.4	44.1	38	38	38
Inflation	All	2.8	1.4	3	3	3
	1740s	5.8	2.5	5.7	5.7	5.8
	1750s	-3	-2.6	-3	-3	-3
	1760s	-1	3.2	-1	-1	-1
	1770s	4.5	1.5	5	5	5
	1780s	7.5	1.8	8.4	8.4	8.4
	1790s	3	3.6	3.1	3.1	3.1
	1800s	3.6	0	3.6	3.6	3.6
Excess expenditure	All		29.1	1.9	1.9	1.9
	1740s		38.1	1.6	1.5	1.5
	1750s		23.8	1.5	1.5	1.5
	1760s		40.7	1.5	1.5	1.5
	1770s		33.7	2.9	2.9	2.9
	1780s		20	2.7	2.7	2.6
	1790s		25.7	1.6	1.6	1.6
	1800s		19.9	1.6	1.6	1.6

Notes: The columns headed “cheap” and fixed basket summarize hypothetical policies where a household only purchases the cheapest crop or a fixed quantity of each to satisfy caloric needs. The last three columns are based on our estimations.

B. Climatic conditions

B.1 Description and specification for tree-ring data

a. General

The data sets containing tree ring growth in Mexico are standardized, but separated into (data) sheets for early and late wood growth measurements for individual trees. Each tree is given an identifier. However, we find multiple instances where this identifier repeats itself in the same data sheet. Hence, multiple observations of a single identifier are erased entirely. The majority of these tree types are Douglas Fir (15x) while the Montezuma Cypress and the Montezuma Pine each have two data sets. Finally, a single data set contains Ponderosa Pine trees. In particular, the Douglas Fir and the Montezuma Cypress, the National Tree of Mexico, are long-lived species that provide reliable and anatomically distinct growth rings (Stahle *et al.*, 2016). The measurement precision for the tree ring growth is 0.001 mm.

b. Preliminary analysis

Fact I: Climatic condition is a granular measure. The right panel of Figure 2 shows 4 forests in the central region of Mexico that are in the vicinity of Mexico city. Eventually, we use 3 forests, as the tree ring growth of the “El Malpaso” forest shows negative co-movement with some of the other 3 forests. In particular, the correlation coefficient of the forest-wide averages of the late tree-ring growth of “El Malpaso” and “Villareal” is -0.161 and significantly different from zero at the 1%

level. Similarly, the correlation coefficient of the forest-wide averages of the early tree-ring growth of "El Malpaso" and "Pinal de Amole" is -0.199 and significantly different from zero at the 1% level. All the other correlation coefficients of the forest-wide averages are positive, and all but two are significant. Unsurprisingly, the two non-significant correlation coefficients involve the "El Malpaso" forest again. Hence, we discard "El Malpaso" as it is, arguably, part of a different local weather pattern. Therefore, we find the total number of trees to be $|\mathbb{I}|$=333. It is noteworthy that Therrell *et al.*, (2006) use the "Cuauhtemoc la Fragua" forest to develop a maize yield variability reconstruction in central Mexico from 1474 to 2001.

Table 6 provides summary statistics for each forest we use in our final specification. Further, we pool all forests in the last column and find 61,250 early and 59,618 late wood growth observations.

Table 6.

Summary statistics for tree ring data sets used in our analysis

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Name	Pinal de Amole		Villareal		Cuauhtemoc la Fragua		pooled	
ID#	33		37		45			
Latitude	20.101		19.315		19.101		19.262	
Longitude	-99.42		-97.525		-97.191		-97.463	
First year	1731		1699		1474		1474	
Last year	1998		2000		2004		2004	
No. of trees	55		109		169		333	
No. of obs	9,301	9,301	16,241	15,975	35,708	34,342	61,250	59,618
Mean of log-growth	6.02	5.66	6.55	6.03	6.42	5.48	6.39	5.66
Std of log-growth-mean	0.01	0.01	0.01	0.01	0	0	0	0
Std of log-growth	0.92	1.04	0.85	1.01	0.72	0.81	0.8	0.94
Std of log-growth-tree-mean	0.42	0.37	0.38	0.5	0.31	0.41	0.42	0.51
	early	late	early	late	early	late	early	late

Notes: See the text for explanations and data sources.

Fact II: Trees grow more in the first half of the year than in the second half. The average log growth is 6.39 (5.66) in the early (late) wood growth sample, respectively. The corresponding standard deviation of the mean is 0.0032 (0.0038). This is in line with results from dendrochronological studies. For example, Stahle *et al.*, (2016) highlight that the main moisture signal occurs during the winter, spring and early summer.

Fact III: The growth pattern of tree rings is, in part, determined by tree-specific fixed-effects. Some trees persistently grow more than others, which resonates with the idea that a statistical tree is subject to fixed-effects. Initial evidence can be found in Table 6. The standard deviation of the tree-specific log-growth in the pooled sample is more than half of the overall standard deviation for both, early and late wood growth. To flesh out this insight, we (separately) regress late and early wood (log-) growth on tree-specific dummies. The corresponding measures of determination, R^2, are 0.22 and 0.27. A joint regression where the same tree-specific dummy explains both, growth in the first and second half of the year, still yields an R^2 of 0.16. Including a single dummy that is equal to 1 for early wood growth, and 0 for late wood growth increases R^2 to 0.37. Hence, idiosyncratic effects alone explain a lot of the total variation, which suggests the use of panel data models to reduce noise and, because the panel data is unbalanced, an omitted variable bias.

Fact IV: The age of a tree (statistically) determines tree ring growth. Age decreases early wood growth linearly, and late wood growth can be explained by a polynomial function of age. We pool observations and discard observations where a tree is older than 400 years. Figure 6 shows the age-specific average (red) and the 5- and 95-percentiles (blue) of the distribution for early-wood (left) and late-wood (log-) growth (right). A simple linear fit explains the mean growth quite well for early wood, and a polynomial of the 8th degree is chosen by the Akaike Information Criterium. While it is straightforward to control for the age of trees in order to reduce noise,[27] it also points to a selection bias. Arguably, years with favorable climatic conditions allow young trees to grow more rapidly and survive a potentially vulnerable time as small saplings. This leads to their selection in the sample. If climatic conditions are persistent, then we would not only observe strong growth in all trees but also more young trees that grow more rapidly because of their age. Hence, we control for the age of the observation in the regression that elicits the time-effects. In particular, we use a linear effect for early-wood growth and let the Bayesian Information Criterion (BIC) decide on the order of the polynomial for late-wood. We employ the BIC (rather than the Akaike Information Criterion) as it penalizes additional coefficients more and we aim to keep the econometric specification parsimonious.

[27] The R^2 for early-wood and late-wood (log-) growth is 0.076 and 0.052, respectively.

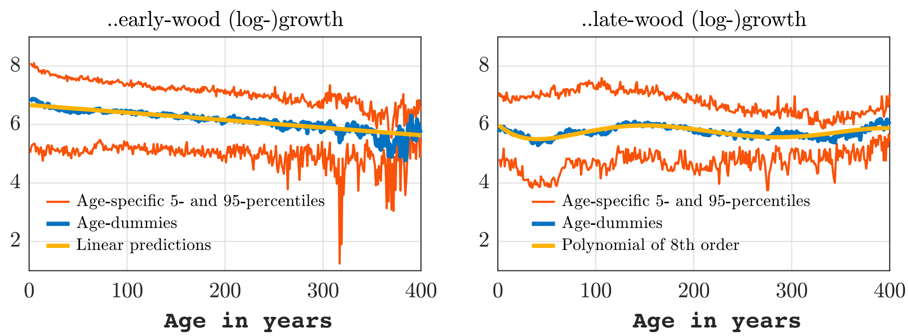

Figure 6. The first 400 years of...

Notes: Age-specific average (log-) growth (red) and the 5- and 95-percentiles (blue) of the distribution for early-wood (left) and late-wood (right).The yellow line represents a prediction using a model that is linear in age (left) and that uses a polynomial of the 8th order. Neither fixed-effects nor time-effects are used.

c. Details on the econometric results

Table 7 summarizes the results for fixed-effects styled regression for Q∈{1,2,3,4} with a static specification (V=0) in the first four columns. We only include observations from 1720 to 1815 as this covers our time of interest. We discard other observations due to the sheer size of the data set. We do not estimate using the random-effects assumption. Such estimates are not robust against a correlation of the (unobservable) tree-specific fixed effects with the (observable) regressors as mentioned above. The fixed-effects specification is robust to this problem. Further, we do not report the results for the 190 time-effects in Table 7. The Akaike information criterion selects a second-order polynomial (Q=2) for the effect of age on late wood growth. Hence, column 2 in Table 7 presents the preferred model among the static specifications.

Table 7.

Panel data regression for tree-ring (log-) growth

Specification	Static				Dynamic
Q	1	2	3	4	1
V	0	0	0	0	2
K (instrument lag)					3
No. of regressors	162	163	164	165	164
Observations	20,596	20,596	20,596	20,596	39,485
Trees	187	187	187	187	182
BIC	-16,900	-16,915	-16,911	-16,901	-44,471
R^2_{within}	0.31	0.31	0.31	0.31	0.53
$R^2_{overall}$	0.53	0.53	0.53	0.53	0.67
Hansen test statistic					0.83
Hansen p-value					0.66
β	-0.0061*** (0.0000)	-0.0057*** (0.0000)	-0.0060*** (0.0000)	-0.0060*** (0.0000)	-0.0004*** (0.0001)
ψ_1	-0.0082*** (0.0000)	-0.0098*** (0.0000)	-0.0082*** (0.0000)	-0.0079*** (0.0000)	-0.0006*** (0.0001)
γ_2		0.0000*** (0.0000)	-0.0000*** (0.0000)	-0.0000*** (0.0000)	
γ_3			0.0000*** (0.0000)	0.0000*** (0.0000)	
γ_4				-0.0000*** (0.0000)	
ω_1					-0.0786*** (0.0164)
ω_2					0.8697*** (0.0170)

Notes: Results from regressions using variations of model (5). See the text for variables and data sources; p-value *** p<0.01, ** p<0.05 and * p<0.1.

What motivates the auto-regressive component in equation (5)? There are several possible explanations that are not mutually exclusive. First, the auto-regressive formulation approximates the age-profile better than a polynomial approximation. Second, the climate that determines climatic conditions is more granular than the area spanned by the three forests. In their accompanying study about the Mexican Drought Atlas, Stahle *et al.* (2016) emphasize that intense "All Mexico" droughts have been rare over the last 600 years. If the local climate is serially correlated the auto-regression captures a local (forest-wide) process and frees up the estimates of the (regional) time-effects from such noise. Thirdly, trees simply have good years and bad years, which can be attributable to health conditions. Either way, the inclusion of the auto-regressive component improves the statistical fit and reduces the background noise.

Dynamic panel data regression introduces the Nickell bias (1981) for the autoregressive coefficients. The Nickell bias renders the lagged regressors endogenous and invalidates an estimation based on a fixed-effects transformation. Lagged observations can serve as instruments but this requires that the error terms are not serially correlated. Arellano and Bond (1991) present a post-estimation test. Unfortunately, it requires a plugin that can turn the standard deviation negative. Fortunately, a small-scale simulation study confirms that a Hansen-style test suffices to pick up auto-correlation between instrumenting lagged endogenous variables and the error term when the sample size is similar to ours. Hence, we use a Hansen-style test to select (or reject) instrumental variables. In accordance with the Hansen-style test, we move the instruments K periods into the past to avoid the serial correlation issue just described.

We use first-differences and a transformation known as orthogonal deviations to estimate the dynamic specification of equation (5) where $V > 0$.[28] We instrument transformed variables with levels and levels with first-differences. The latter expansion is known as system-GMM. We set $V \in [2,8]$ to allow for up to 4 years of auto-regressive components predicting the current growth. Hence, the Bayesian information criterion chooses our preferred model among 112 perturbations of equation (5).[29]

The preferred specification is reported in the last column of Table 7. It employs $V = 2$ auto-regressive lags and a linear late-wood age model ($Q = 1$). Three additional auto-regressive terms are used as instruments. The Hansen test suggests removing two lags. The information criterion selects the orthogonal-deviation transformation. Further, the estimation employs both, the difference and level equations in the regression (system-GMM). The use of the system-GMM also explains why the number of observations almost doubles.

Figure 7 plots the estimated time-effects, which serve as a measure for the climatic conditions F, for the static (top, $V = 0$) and the dynamic specification (bottom, $V > 0$) against the unconditional mean (log-) growth for both, early-wood (left) and late-wood (right). The static model picks up some differences. For example, early-wood growth from the static time-effect estimation appears much lower around 1740 while it corrects downward at the beginning of the 19th century. Similarly, late-wood growth from the static estimation is generally below (above) the unconditional mean before (after) 1775. Nonetheless, the estimated time-effects follow the unconditional means quite closely. The co-movement between the unconditional mean and the time-effects in the dynamic estimation are less systematic. We believe that several effects can create persistence in the measurements but are controlled for with the auto-regressive specification drop, which leads to the less-smooth display of the time-effects.

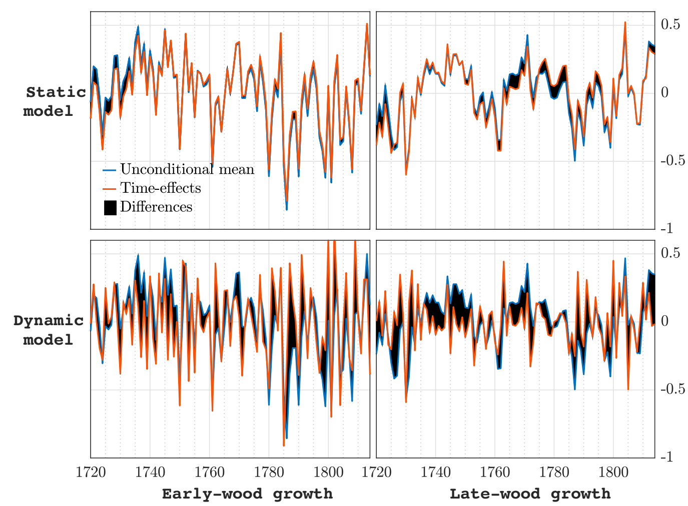

Figure 7. Unconditional mean growth vs. point estimates of time-effects

Notes: Comparison of (re-centered) unconditional mean growth (blue line) for a given year with the (re-centered) estimated time-effects (red line) from the static model (top panel) and the dynamic model (bottom panel).

d. Details on the construction of the Markov transition matrix

We define the early (late) climatic conditions F_w (F_m) in year t which determine the wheat (maize) yield as the time-effects δ_t estimated by the dynamic panel regression specification, or $F_w = \delta_{e,t}$ and $F_m = \delta_{l,t}$ where e (l) indicates the early (late) wood growth. Next, we discretize F by grouping realizations into distinct bins, as proposed by Tauchen (1986). We divide early F_w and late F_m climatic conditions into groups using the k-means algorithm. The silhouette method determines the optimal number of clusters with a possible range from 4 to 8. The optimal choices are 4 and 5 for the early and late climatic conditions, respectively. Hence, the early period has the possible state realizations $W = \{W_1, W_2, W_3, W_4\}$ and the late period has the state realizations $M = \{M_1, M_2, M_3, M_4, M_5\}$.

The two top panels of Figure 8 characterize these groupings and their relative occurrence for the early and late climatic conditions on the left and right, respectively. The mid-points are the means of the group members and represent the discretized state realizations of the seasonal climatic conditions. The numeric realizations can be found underneath the two top panels of Figure 8. The bin edges are defined by the nearest mid-points.

[28] The dynamic structure in equation (5) invalidates an estimation based on the fixed-effects transformation because of the the Nickell bias.

[29] We have $|V|$=7 and $|Q|$=4 which provides us with 28 perturbations of the model represented by equation (5). Further, there are two different transformations, namely first differences and orthogonal deviations, which lead to 56 perturbations. Then we can either regress the transformed variable on levels alone or employ a full-fledged system-GMM estimation. Hence, there are 112 perturbations in total.

The central panel shows the time series of discrete climatic conditions for early and late periods in blue and red, respectively. Finally, the two bottom panels show the scatter plot of original climatic conditions using a grid that corresponds to the bins derived above. The left (right) bottom panel has the early (late) period on the horizontal axis and the subsequent late (early) period on the vertical axis.

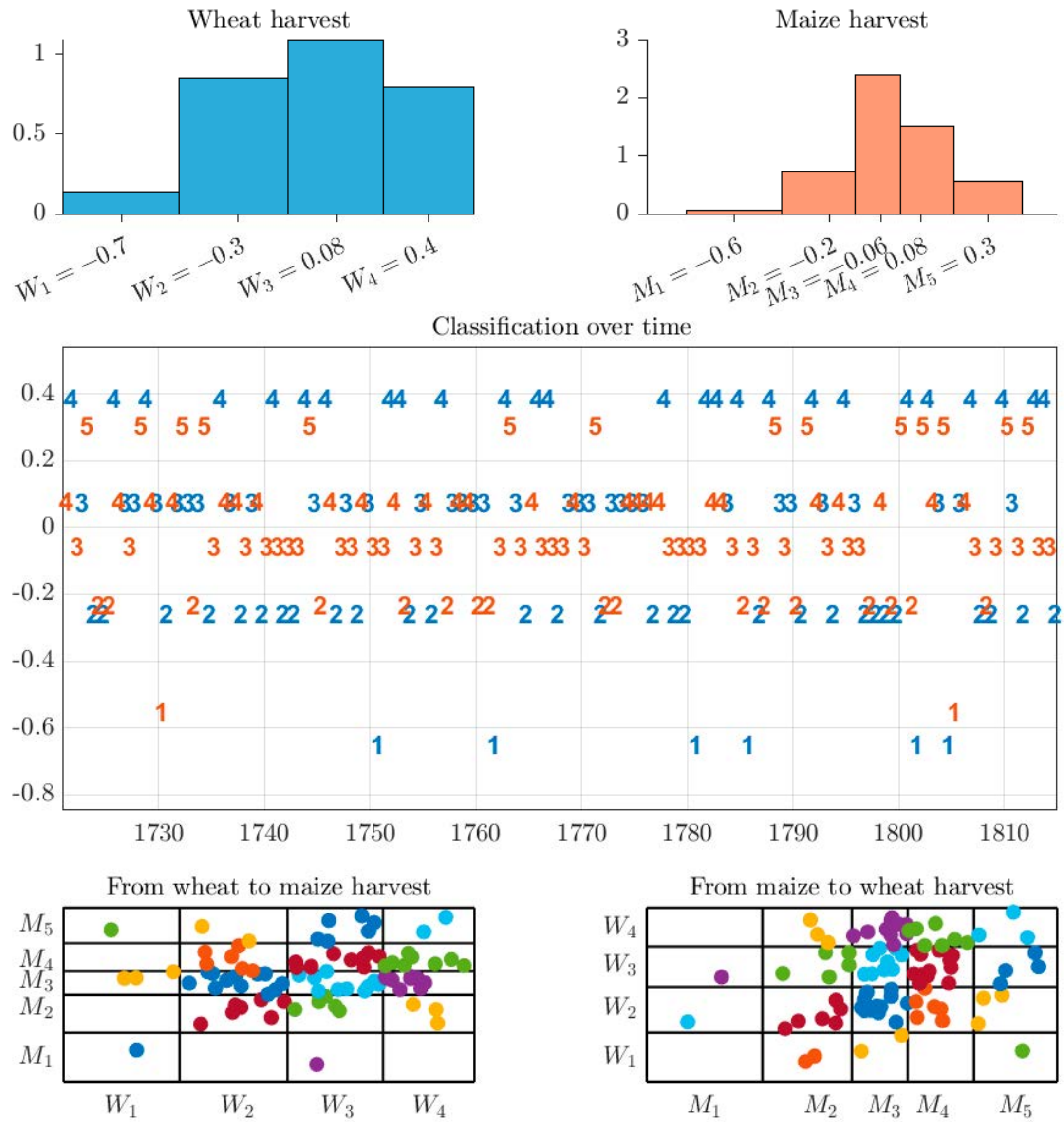

Figure 8. Discrete climatic conditions, time series, and Markov transition matrix.

Notes: The two top panels show the density of climatic condition realizations for the early and late half years when the wheat and the maize harvest are determined. The central panel shows the time-series of discretized climatic conditions using numerical identifiers for each group of climatic conditions. The two bottom panels show the scatter plot of climatic conditions. The left (right) bottom panel has the early (late) period on the horizontal axis and the subsequent late (early) period on the vertical axis.

The Markov transition matrix $\prod_{W2M}$ describes the probabilities of transitioning from any state in the early period to any state in the late period. In particular, the element $\pi_{W,M}$ in row *W* and column *M* describes the probability to reach state *M* in the next period when the current state is *W*. We require $\pi_{W,M} \geq 0 \ \forall \{W,M\}$ and $\sum_{M=1}^{5} \pi_{W,M} = 1$. Similarly, the transition matrix describing the transition probability of going from state *M* to *W* is denoted $\prod_{M2W}$. The elements and restrictions of $\prod_{M2W}$ correspond to those of $\prod_{W2M}$. This discretization allows us to rewrite equations (2) and (3) as equations (7) and (6), respectively.

In order to determine a smooth Markov transition matrix, we fit a bivariate normal distribution using Maximum-likelihood without any restriction on the covariance. The resulting transition matrices can be found in Table 8.

Table 8.
Markov transition matrix

From wheat to maize harvest					
	M_1	***M_2***	***M_3***	***M_4***	***M_5***
W_1	0.063	0.37	0.32	0.2	0.042
W_2	0.031	0.28	0.33	0.28	0.082
W_3	0.015	0.20	0.31	0.34	0.14
W_4	0.007	0.13	0.27	0.38	0.21

From maize to wheat harvest				
	W_1	***W_2***	***W_3***	***W_4***
M_1	0.034	0.23	0.43	0.31
M_2	0.049	0.28	0.42	0.25
M_3	0.065	0.31	0.41	0.22
M_4	0.082	0.34	0.39	0.18
M_5	0.110	0.37	0.37	0.15

Notes: Transition probabilities used in estimation. Rows sum to one.