Wood anatomy and stress distribution in the stem of *Pinus pinaster* Ait.

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SUMMARY

A 70-year-old *Pinus pinaster* Ait. tree growing in a Spanish forest was cut down and the structure and shape of its crown and trunk were studied. The distribution of longitudinal stresses in the stem due to its self-weight and several wind loading were calculated using the structural theory of a cantilever beam. Wood anatomical features related to its mechanical behaviour (tracheid length, wood density, and heartwood area) were determined in 25 stem slices along the length of the trunk.

The longitudinal distribution of stress along the trunk had a peak at 1/4 of the total tree height, and this trend was similar to those of the anatomical features: their maximum values occurred at the same height. This could mean that the tree reinforces its support structure at the point of highest failure risk, where fractures can occur.

KEY WORDS: Biomechanics, Tracheid length, Wind loading, Stress, *Pinus pinaster* Ait.

INTRODUCTION

The trunk of a tree has a specialised structure in order to support mechanical efforts, due to the self-weight of the tree (crown and stem) and to the external loads (wind, snow). Wood structure, considered as a strengthening tissue, is supposed to be closely related to
the stress level which affects it during the life of the tree. Although the relationship between wind loading and tree form has been studied (Mattheck, 1991), a detailed understanding of the effect of wind loading and tree weight on the internal wood structure has not been developed yet (Gardiner, pers. com.).

Several authors have used the structural theory of cantilever beams in order to analyse bending of trunks and branches: as referred by Gardiner (1991), simple relationships have been developed to explain the static behaviour of the tree stem and branches (Leiser y Kemper, 1973; Petty y Worrel, 1981; Mamada et al., 1984; Castéra y Morlier, 1991), and these relationships seem to work well. Nevertheless, complete theories to describe large deflections are available (Morgan y Cannell, 1987; Milne y Blackburn, 1989). Consideration of the tree as a forced damped harmonic oscillator, has allowed to model its dynamic response to wind loading (Mayer, 1987; Gardiner, 1991; Gardiner, 1993), but these considerations are not going to be observed in this work, as well as those related to growth stresses (Archer, 1986). Both Leiser y Kemper (1973) and Milne y Blackburn (1989) have found that axial stresses due to wind loading vary along the stem with a maximum occurring at a position which depends on taper. According to Mamada et al. (1984), the theoretical height of maximum stress was in good agreement with the height at which the stem breaks. However, other authors (Petty y Swain, 1985; Mattheck, 1991; Wood, 1995) suggest that the stress should be constant along the stem.

Here we present a mathematical model of the bending of the main stem of a conifer using the mechanical static theory for cantilever beams, as described by the authors above. Distribution of axial stresses along the stem considering different wind forces is studied, and results are discussed in relation to those authors who suggest that the stress is constant along the stem.

Specific gravity, fibre length, microfibrillar angle (MFA) and grain characteristics are some of the structural parameters of the wood related with its mechanical behaviour (Larson, 1969). The relationship between fibre length and stiffness is not fully understood (Zobel y Jett, 1995), as it is also considered that the stiffness depends on wall thickness and size of a cell.Megraw (1985) concluded that fibre length is strongly and inversely correlated with the MFA angle, large angles corresponding to short tracheids. Moreover, MFA seems to be closely related with the mechanical properties of the log (Boyd y Foster, 1974; Wainwright et al., 1976; Megraw, 1985), flat angles (i.e., large fibres) corresponding to great stiffness. From the results obtained by several authors and described in Megraw (1985) and Zobel y Van Buijtenen (1989) it is deduced that vertical variations of fibre length within the tree follows some well known trends. One of these is that fibre length increases with increasing height up to a maximum level some meters above the base, and then decreases towards the top.

Wood density, being mainly a combination of proportion of latewood, wall thickness and cell size (Zobel y Van Buijtenen, 1989), is a complex characteristic, and the results of different authors about its vertical distribution along the stem are quite different.

Regarding the heartwood, Hillis (1987) showed that it usually begins its formation at a height between 1-3 m above ground level, and in this cross-section the tree has the greatest percentage area of heartwood.

In this paper, besides the mechanical analysis, we study the variation along the stem of three anatomical features: tracheid length, wood density and heartwood relative area, and the results are compared to those obtained for the distribution of stress. It looks reasonable to think that the tree would adapt its stem’s characteristics in order to support the
efforts at which it is going to be exposed because of the action of the wind. So it would reinforce the stem in those parts in which stresses are going to be greater. This paper analyse this by comparing both mechanical and anatomical analyses.

MATERIAL AND METHODS

Material

The tree chosen for this work was a *Pinus pinaster* Ait. from the Sierra de Gredos (Central Spain). It was selected because of its shape and upright stem, in a flat site (10% slope), in order to avoid as far as possible the presence of compression wood. The stand was a natural forest community at 550 m altitude, and there were 470 trees per ha. The tree was 70-year-old and 20.3 m high, with its canopy beginning at 15.5 m. The base diameter was of 26.4 cm, and the breast-height diameter was of 23 cm.

It is necessary to point out that only one tree has been sampled, and therefore the conclusions can only be indicative and cannot be tested statistically between trees.

Methods

a) Mechanical theory

The development of the mechanical model is schematically presented in Figure 1. The main limitations of the adopted model are that it does not account for tree deflection or for dynamic effects, and that growth stresses are not considered neither. The hypothesis on which the mechanical analysis is based are summarised in the following lines, being the ones usually adopted by several authors who have studied the bending of tree trunks and branches (Leiser y Kemper, 1973; Mayer, 1987; Milne y Balckburn, 1989; Gardiner, 1991).

a) The stem of standing trees can be treated as an elastic cantilever beam, rigidly fixed on one side and free on the other. Its section varies with height, and this non-uniform taper can be described by a mathematical function:

\[ r_i = a \cdot L_i^b \]  

where \( r_i \) is the radius of the i-section, \( L_i = h-h_i \) is the distance from the i-section to the top of the stem and a and b parameters to be estimated; this expression is similar to the one described by Gardiner (1991).

b) The transverse section of the stem is considered circular, with an area \( S_i = \pi \cdot r_i^2 \) and a second moment of area \( I_i = (\pi \cdot r_i^4) / 4 \).

c) In order to calculate the self-weight of the tree, its canopy weight can be evaluated as a point vertical force applied in its centre of gravity.

d) In order to calculate the wind load, a horizontal point load applied also in the canopy centre of gravity can substitute it.

e) When bending, trees will usually fail on the compression side first, because wood is an extremely anisotropy material whose compression strength is about half the tensile
strength (Mossbrugger, 1990). In the development of the method the most unfavourable case will always be considered, searching for the point where maximal compression tensions occur.

Construction of the model. The weight of the tree is divided into stem weight and canopy weight. As for the stem load, each section of the trunk is at any time supporting the weight of the portion of trunk that stands over. Using equation (1), which ties radius to height, and applying the concept of solid of revolution, we obtain

\[ V_i = S_i \cdot L_i / (2b+1) \]

which gives us the stem volume supported by the i-section. According to this the related stress in the i-section is

\[ \sigma_{psi} = P_{si} / S_i = V_i \cdot \rho / S_i = L_i \cdot \rho / (2b+1) \]

where \( \rho \) is the wood fresh density, and \( P_{si} \) is the weight of the portion of the stem which remains over the transverse section of area \( S_i \).

As mentioned above, the canopy weight \( P_c \) is applied as a point load in the centre of gravity \( G \) of the crown (Fig. 1), generating constant axial stresses like...
Usually, the centre of gravity of the crown will be eccentric, and the distance to stem
$e$, horizontal angle $\theta$, and height $h_g$ can define its situation. The effect of this eccentricity
was studied by Peltola y Kellomaki (1993): the eccentric load induces a bending moment
$M_c = P_c \cdot e$, which is constant along the stem. Stresses due to bending change in each
cross-section from tensile at the convex side to compressive at the concave side of the
element, and maximum values are reached in the periphery:

$$\sigma_{Mc} = M_c \cdot \frac{r_i}{I_i}$$

Assuming that wind force is effective on the centre of gravity of the crown, the bending
moment due to wind flow varies with the height of the cross-section considered. Its
value can be calculated by $M_{w} = F_{we} \cdot h'_i$, where $h'_i = h_g - h_i$ is the distance from the
crown centre of gravity to the i-section, and $F_{we}$ is the effective wind force on the crown.
This bending moment generates a family of stresses according to (4). $F_{we}$ can be calculated (Mayer, 1987) by using:

$$F_{we} = C_d \cdot d \cdot A_c \cdot \frac{u^2}{2}$$

where $C_d$ is the drag coefficient as defined by Mayhead (1973), $d$ is the air density, $A_c$ is
the projection area of the crown perpendicular to the air flow, and $u$ is the velocity of
wind flow.

The most unfavourable case is to be considered, which means that wind flows in such
a direction that compressive stresses due to wind add to the compressive stresses due to
crown eccentricity.

Finally, the maximal compressive stress in the i-cross-section of the stem adopts the
expression

$$\sigma_i = \sigma_{Pe} + \sigma_{Pc} + \sigma_{Mc} + \sigma_{Mw}$$

$$r_i = \frac{\rho}{2b + 1} \left( \frac{h_g - h_i}{r_i} \right)$$

where all the terms are already known from equations (3), (4), (5) and (6).

**Determination of mechanical parameters.** The orientation and insert angle of all the
principal branches were determined in the standing tree; then it was cut down and
delimb. The insert diameter, length and total weight were measured for each principal
branch, as well as the weight of secondary branches and that of the previously separated
foliage. Whorls height in the stem were measured and twenty-five slices of stem were cut,
each one in the middle of an inter-whorl. Total height ($h_t$) was measured with tape mea-
sure on the laying tree. Crown weight ($P_c$) was worked out by adding up the partial
weights of all branches. Green density ($\rho$) was obtained by averaging the densities calcu-
lated for each stem slice (by dividing weight per volume). Using data of weight, length
and situation (orientation and insert angles) of each principal branch and applying the
usual procedure in mechanics, the position of the crown’s centre of gravity was worked
out with respect to a Cartesian reference system with its origin in the centre of the tree base. That position was thus defined by co-ordinates \((x_g, y_g, z_g)\); height \(h_g\) is \(z_g\); and crown eccentricity \((\epsilon)\) is \(\epsilon = (x_g + y_g)^{1/2}\). After determining the position of each branch top, crown vertical projection was graphically represented, and its area \(A_c\) measured with graphic analyser Delta-T System Image Analyser (DIAS).

Using potential regression techniques to data of radii and heights of the cross-sections, a mathematical expression like (1), describing stem taper, is obtained.

As regards to the drag coefficient \(C_d\), Mayhead (1973) showed that its value depends on species, and varies as a function of the wind flow velocity, decreasing when this increases. This variation is non-uniform, but being of small importance for high speeds, it could be considered as a constant value for \(u > 30\) m/s. Fraser (1964) gave a relationship between drag force \(F_{we}\) and wind velocity and tree weight, and Landsberg y Jarvis (1973) proposed several expressions which give \(C_d\) values dependant on leaf areas. In this case Mayhead’s results are chosen: characteristic \(C_d\) values for the principal forest species of the British Islands are determined in the referred work; from the values assigned to several pines \((P. sylvestris, P. nigra, P. contorta)\) and considering their foliage features, the following \(C_d\) values for \(P. pinaster\) can be assumed: 0.85 for 5 m/s; 0.80 for 7.5 m/s; 0.70 for 10 m/s; 0.45 for 15 m/s; 0.40 for 17.5 m/s; 0.35 for 20 m/s.

Wind flow generates a very complex load, which has been studied in detail by other authors such as Gardiner (1995). Real winds are not going to be considered here, so wind flow speed \(u\) is a constant in the model. A value that generates stresses next to buckling will be chosen. Thus, this wind velocity will not be a real one, but will be considered for the purpose of this study as the usual maximum wind velocity for this region. On the other hand, wind is not constant nor static but variable and fluctuating: the peak wind loading can be almost ten times higher than the mean wind depending on the density of the stand. As explained above, these phenomena are not going to be considered.

Finally, stress depends only on section radius or height of the slice.

b) **Anatomical features**

*Tracheid length.* Wooden cubes (8 cm³) were cut from the periphery of each stem slice in the radial direction defined by crown eccentricity (the most unfavourable in case of bending). After twenty-four hours in water, small longitudinal slices of the last-but-one early wood ring were cut. Then plunging them in a solution of glacial acetic acid and hydrogen peroxide 30 % in equal parts in a stove at 60 °C for 30 days (the time required to allow easy separation of tracheids) macerated these ribbons. Once macerated, the material was observed with a *Visopan Reichert* microscope. At least forty tracheids in each sample were drawn and their length measured with an image analyser, taking care that only uncut cells were measured (Zobel y Jett, 1995).

*Heartwood.* The total and heartwood areas of every stem cross-section were measured using an image analyser. The relationship between both features is the relative heartwood cross-section area.

*Density.* Density was measured in terms of dry weight per volume for the wooden cubes referred above, just before early wood was cut away.
RESULTS

a) Mechanical analysis

The following parameters were obtained directly by measuring the tree or its samples as described above:

\[ P_e = 99 \text{ kg} \quad h_i = 20.3 \text{ m} \quad h_g = 17.8 \text{ m} \quad e = 48 \text{ cm} \]

\[ \theta = 62.7^\circ \quad \rho = 1.23 \text{ g/cm}^3 \]

The most accurate expression (1) that represented the stem taper was \( r_i = 26.38 \cdot L_i^{0.3} \) (i.e. \( a = 26.38 \) and \( b = 0.5 \)), with \( r = 0.98 \).

An adequate wind velocity is chosen in order to determine the wind force: \( u = 15 \text{ m/s} \), that will generate stresses close to failure. For this velocity and for the considered species, we could take drag coefficient as \( C_d = 0.45 \). Thereby, the values needed to determine the wind effect are

\[ u = 15 \text{ m/s} \quad C_d = 0.45 \quad A_e = 17.5 \text{ m}^2 \quad d = 1.226 \text{ kg/m}^3 \quad F_{we} = 1,086.2 \text{ N} \]

The vertical distribution of maximum compression stresses in the stem is shown in Figure 2. Maximal stress is produced at a height of 5.76 m, and reached \( \sigma_{max} = 19.7 \text{ MPa} \).

b) Anatomical features

The maximum mean value of tracheid length (5.1 mm) appears at a height of 4.07 m (20 % of tree height), while lowest values are those from the upper sections (2.8 mm at 19 m). In the work of Echevarría y De Pedro (1948) on \( P. \ pinaster \), the longest tracheids were found in the youngest annual growth rings, and for a given ring, the tracheid length increased with height up to a maximum at 3-4 m for a total height of 16.3 m (i.e. at 20-25 % of tree height) and then decreased again towards the top. Similar results were obtained by Megraw (1985) for \( P. \ taeda \).

Heartwood forms a sub-stem into the trunk, which is surrounded by sapwood. Relative heartwood area in the cross-sections varies with height: (highest values are over 20 %) and they are obtained also at 4.07 m (20 % of stem height). According to Hillis (1987) heartwood begins its formation within 1-3 m above ground level; the heartwood area in relation to the total section area can also be greater at these levels than at the ground level. According to this, the heartwood, as determined in our tree, tapers from the level of first initiation towards both crown and butt.

Density values present more dispersion, ranging from 0.26 g/cm³ in the tip to a maximum of 0.66 g/cm³ at 5.7 m (28 % of tree height), the tendency being a decrease in density as height increases.

Evidence of compression wood was not found in the analysed slices.
DISCUSSION

The maximum stress value ($\sigma_{\text{max}} = 19.7$ MPa) is less than the compression strength determined by Gutiérrez y Plaza (1967) for standing trees of $P. \text{pinaster}$ ($\sigma_{\text{max}} = 21$ MPa). These values are similar to those obtained by Milne y Blackburn (1989), taking into account the different height of the studied trees. As explained before, the bending moment due to wind $M_w$, varies with height, increasing from the top to the base of the stem. But the cross-section area which supports that force also increases in the same way, so that the maximum stresses will occur at the height in which the increment of resistant area balances the bending moment increment. This will be the height of maximum stress at which the breakage will occur, and will depend on tapering.
Considering winds of enough velocity (those which could produce breakage), the stresses due to the self weight of the tree could be considered worthless as opposed to the wind effect (although deflection due to wind would produce a double effect of increasing the eccentricity of the crown and diminishing the bending height), except for the stresses due to crown eccentricity in the upper parts of the stem, which are just under the centre of gravity and which imply a relative maximum. As predicted by Mattheck (1991) axial compression due to stem and crown weights (less than 0.2 MPa at its maximum) is not as significant as bending. Therefore, the basic distribution pattern of stresses is similar to the one obtained by considering wind stresses only.

A comparison of different results is shown in Figure 3, which presents the vertical distribution of the different features along the stem: maximum axial stresses, tracheid length, heartwood relative area and wood density. All patterns are quite similar: although no statistical regression can be established among data, it is a fact that all variables reach their three largest values at the same height. This height is found to lie between 5.76 m and 4.07 m (i.e. around 5 m); it means about 1/4 (25%) of the total height of the tree (20.3 m) and 1/4 to 1/3 of the bending height (17.7 m). These values are similar to those obtained by Leiser y Kemper (1973) and Milne y Blackburn (1989) for the maximum stresses, and by Echevarría y De Pedro (1948) and Megraw (1985) for the maximum fibre length. Related to density, in most of the cases studied by Okkonen et al. (1972) specific gravity decreased when height increased, and for hard pines as P. pinaster, the density pattern is heavy at the base and lighter at the top (Zobel y Van Buijtenen 1989). Megraw

![Fig. 3.-Vertical distribution along the stem of some features: maximum axial stresses, tracheid length (mean values), heartwood relative area and wood density. Horizontal lines mark the area where maximum values are reached](image-url)
(1985) found the highest values for *P. taeda* in the area extending from the stump to the breast height, decreasing then rather rapidly as height increases up to 5 m. However, none of them reports a height of maximum density like the one for tracheid length.

These results support the idea that the tree should protect itself, reinforcing its structure (with larger tracheids, higher density and more heartwood surface) in those areas where greatest stresses are going to take place and where the reinforcement should be more useful.

In Figures 2 and 4 where the compression stresses are indicated, a secondary peak at approximately 17 m can be observed, which is consequence of having applied the loads at the centre of gravity of the crown.

Several authors (Metzger, 1893; Petty y Swain, 1985; Mattheck, 1991; Wood, 1995) suggest that the stress should be constant in the stem periphery. As Mattheck (1991) demonstrates the relationship between stem diameter and height is defined by the postulate of constant axial stress on the stem surface of trees, these taper off towards the top in order to decrease the wind loads higher up; trees should maintain and restore the state of constant stress by permanent adaptation to the ever-changing external loads, and this leads to the concept of adaptive growth.

The distribution trends of axial stresses (compression) for seven wind speeds (0, 5, 7.5, 10, 15, 17.5 and 20 m/s) in the analysed tree are presented in Figure 4. The forces and stresses have been calculated using the same model described above, taking as drag coef-
coefficients: 0, 0.85, 0.80, 0.70, 0.45, 0.40 and 0.35 respectively. The first (u = 0 m/s) is the case where only the self-weight of the tree is considered, and the shape of the line obeys to the bending moment produced by the eccentric crown. The figure shows that for slow wind speeds (5-10 m/s) the stress can be considered quite constant along the stem, but when wind speeds are higher (> 15 m/s), the peak on the basal log (at 5 m high) is more and more conspicuous. This could mean that surface stress is really constant during the greatest part of the tree-life, when environmental conditions are «normal» (slow wind speeds); but, occasionally, external loads reach higher values because of unusual winds, which can produce rupture or other damages, and in this situation stress is not constant but presents a peak at the basal log. Therefore the tree reinforces its mechanical structure at this point.

On the other hand, it is possible that the strain, and not the stress, is constant in the stem periphery. It is easy to demonstrate (Wilson y Archer 1979) that when an ideal beam of a shape following the «D3 law» (i.e. linear correlation between diameter cubed vs. height) is bent, the maximum strains along the external surface of the beam are constant along its whole length. Real trees do not often fit mathematical shape expressions closely, and there are differences in the type of parabolic followed by different species. Anyway, in the opinion of these authors what is really important is the fact that, as trees grow, they tend to maintain a shape that keeps the maximum strains along the stem constant, because of the existence of a feed-back mechanism between the strain levels at cells in the cambial zone and the rate of cell production and wood formation.

But the constant strain in surface is compatible with a non-constant stress distribution depending on Young’s modulus (E). It is well known (Cannell y Morgan, 1987) that Young’s modulus increases with specific gravity; as a matter of fact, in their study, specific gravity accounted just for 53 % of the variation in E among branch samples. Hooke’s law (strain = stress/E) means that the greater the value of E, the lower the strain produced by the stress applied. The relationship between stresses and wood density is presented in Figure 3, and correlation between both distribution trends was mentioned above. When stress has a peak, wood density presents its greatest values too, and the same is expectable for Young’s modulus: this should mean that strains would be lower at that height for the same stress applied, i.e., strains at the height of maximum stress (and maximum specific gravity) could be similar to those at the heights where stresses are lower. This would mean a support for the constant-strain hypothesis.

Growth stresses (Archer, 1986) have not been considered in this analysis, although they are very important in providing mechanical rigidity and in reducing the compression stresses on the downside of the bent tree. They act like a tensile pre-stress (Mossbrugger, 1990) which implies an enlargement of compressive strength because of a decrease of the effective stress supported. The value of the growth stresses for coniferous trees like this, is not usually over 7 MPa (Fournier et al., 1990), which represents, in the case of the analysed tree, that failure will occur at u = 20 m/s instead at u = 16 m/s. In any case, this variation in the strength does not have any influence in the shape of the vertical distribution of axial stresses. Nevertheless, growth stresses could have some importance on the formation of heartwood, since usually stems are in tension in the outer rings and in compression in the inner ones, where heartwood is formed.

Another aspect, which has not been considered in this work, is the additional contribution of the overhanging crown in bending. As referred by Gardiner et al. (1997), this could underestimate the stress by up to 20 %, depending on the weight of the crown, but it
does not seem to have a further influence on the distribution pattern of these stresses either.

The results obtained in this paper could be of some importance in order to model the growth of conifers and the formation of its woody tissue according to winds. If the trees that are more exposed to the wind have stronger wood (i.e. larger tracheids), this can be useful to produce stronger wood by regulating the density of stands and, thus, the effective wind velocity.

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RESUMEN

Anatomía de la madera y distribución de tensiones en el tronco de *Pinus pinaster* Ait.

Un ejemplar de 70 años de edad de *Pinus pinaster* Ait. en Arenas de San Pedro (Ávila) fue apeado, y se procedió a estudiar la estructura y forma de su fuste y de su copa. La distribución a lo largo del fuste de las tensiones longitudinales debidas tanto al peso propio del árbol como a diversas cargas de viento se estudió mediante la aplicación de la teoría de voladizos del cálculo de estructuras estático. Se obtuvieron 25 rodajas correspondientes a otras tantas secciones transversales a lo largo del fuste, en las cuales se estudiaron diversas características anatómicas de la madera relacionadas con su comportamiento mecánico (longitud de las traqueidas, densidad y superficie de duramen en la sección).

La distribución de tensiones axiales a lo largo del fuste presenta un pico a una altura sobre el suelo de ¼ de la total del árbol, y el patrón de distribución seguido por esta variable es similar al que presentan las características anatómicas; los valores máximos de todas ellas se presentan en el mismo intervalo de alturas. Esto podría significar que el árbol refuerza su estructura de sostén en la zona de máximo riesgo de rotura, donde se alcanzan las mayores tensiones y se puede producir el fallo de la estructura.

PALABRAS CLAVE:

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Empuje del viento
Tensión
Estrés mecánico

*Pinus pinaster* Ait.

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